References

1 Main Paper:

2 Extensions:

   1 Non-binary treatments: “Covariate Balancing Propensity Score for General Treatment Regimes.” working paper
   

3 Software: *CBPS: R Package for Covariate Balancing Propensity Score* available for download at the CRAN

These and other related materials available at [http://imai.princeton.edu](http://imai.princeton.edu)
Causal inference is a central goal of scientific research

Randomized experiments are not always possible
→ Causal inference in observational studies

Experiments often lack external validity
→ Need to generalize experimental results to a target population

Importance of statistical methods to adjust for confounding factors
Distinction between observed and unobserved confounders
Overview of the Talk

1. **Review:** Propensity score
   - propensity score is a covariate balancing score
   - matching and weighting methods

2. **Problem:** Propensity score tautology
   - sensitivity to model misspecification
   - adhoc specification searches

3. **Solution:** Covariate balancing propensity score (CBPS)
   - Estimate propensity score so that covariate balance is optimized

4. **Evidence:** Reanalysis of two prominent critiques
   - Improved performance of propensity score weighting and matching

5. **Software:** R package **CBPS**

6. **Extension:** Non-binary treatments
Propensity Score

- Setup:
  - $T_i \in \{0, 1\}$: binary treatment
  - $X_i$: pre-treatment covariates
  - $(Y_i(1), Y_i(0))$: potential outcomes
  - $Y_i = Y_i(T_i)$: observed outcomes

- Definition: conditional probability of treatment assignment

$$\pi(X_i) = \Pr(T_i = 1 \mid X_i)$$

- Balancing property (without assumption):

$$T_i \perp \!
\!
\!
\perp X_i \mid \pi(X_i)$$
Rosenbaum and Rubin (1983)

- Assumptions:
  1. Overlap:
     \[ 0 < \pi(X_i) < 1 \]
  2. Unconfoundedness:
     \[ \{ Y_i(1), Y_i(0) \} \perp T_i \mid X_i \]

- Propensity score as a dimension reduction tool:
  \[ \{ Y_i(1), Y_i(0) \} \perp T_i \mid \pi(X_i) \]

- But, propensity score must be estimated (more on this later)
Use of Propensity Score for Causal Inference

- Matching
- Subclassification
- Weighting (Horvitz-Thompson):

\[
\frac{1}{n} \sum_{i=1}^{n} \left\{ \frac{T_i Y_i}{\hat{\pi}(X_i)} - \frac{(1 - T_i) Y_i}{1 - \hat{\pi}(X_i)} \right\}
\]

where weights are often normalized

- Doubly-robust estimators (Robins et al.):

\[
\frac{1}{n} \sum_{i=1}^{n} \left[ \left\{ \hat{\mu}(1, X_i) + \frac{T_i (Y_i - \hat{\mu}(1, X_i))}{\hat{\pi}(X_i)} \right\} - \left\{ \hat{\mu}(0, X_i) + \frac{(1 - T_i) (Y_i - \hat{\mu}(0, X_i))}{1 - \hat{\pi}(X_i)} \right\} \right]
\]

- They have become standard tools for applied researchers
Propensity Score Tautology

- Propensity score is unknown
- Dimension reduction is purely theoretical: must model $T_i$ given $X_i$

- Diagnostics: covariate balance checking
- In practice, adhoc specification searches are conducted
- **Misspecification** is possible especially for non-binary treatments

- Theory (Rubin *et al.*): ellipsoidal covariate distributions
  $\leadsto$ equal percent bias reduction
- Skewed covariates are common in applied settings
- Propensity score methods can be sensitive to misspecification
Simulation study: the deteriorating performance of propensity score weighting methods when the model is misspecified

**Setup:**
- 4 covariates $X_i^*$: all are \(i.i.d\). standard normal
- Outcome model: linear model
- Propensity score model: logistic model with linear predictors
- Misspecification induced by measurement error:
  - $X_{i1} = \exp(X_{i1}^*/2)$
  - $X_{i2} = X_{i2}^*/(1 + \exp(X_{i1}^*) + 10)$
  - $X_{i3} = (X_{i1}^* X_{i3}^*/25 + 0.6)^3$
  - $X_{i4} = (X_{i1}^* + X_{i4}^* + 20)^2$

Weighting estimators to be evaluated:
1. Horvitz-Thompson
2. Inverse-probability weighting with normalized weights
3. Weighted least squares regression
4. Doubly-robust least squares regression
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Weighting Estimators are Sensitive to Misspecification

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Smith and Todd (2005, *J. of Econometrics*)

- LaLonde (1986; *Amer. Econ. Rev.*):
  - Randomized evaluation of a job training program
  - Replace experimental control group with another non-treated group
  - Current Population Survey and Panel Study for Income Dynamics
  - Many evaluation estimators didn’t recover experimental benchmark

- Dehejia and Wahba (1999; *J. of Amer. Stat. Assoc.*):
  - Apply propensity score matching
  - Estimates are close to the experimental benchmark

- Smith and Todd (2005):
  - Dehejia & Wahba (DW)’s results are sensitive to model specification
  - They are also sensitive to the selection of comparison sample
Propensity Score Matching Fails Miserably

- One of the most difficult scenarios identified by Smith and Todd:
  - LaLonde experimental sample rather than DW sample
  - Experimental estimate: $886 (s.e. = 488)
  - PSID sample rather than CPS sample

- **Evaluation bias:**
  - Conditional probability of being in the experimental sample
  - Comparison between experimental control group and PSID sample
  - “True” estimate = 0
  - Logistic regression for propensity score
  - One-to-one nearest neighbor matching with replacement

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**Covariate Balancing Propensity Score**

- **Idea:** Estimate the propensity score such that covariate balance is optimized

- **Covariate balancing condition:**

\[
\mathbb{E}\left\{ \frac{T_i \tilde{X}_i}{\pi_\beta(X_i)} - \frac{(1 - T_i)\tilde{X}_i}{1 - \pi_\beta(X_i)} \right\} = 0
\]

where \( \tilde{X}_i = f(X_i) \) is any vector-valued function

- **Score condition** from maximum likelihood:

\[
\mathbb{E}\left\{ \frac{T_i \pi'_\beta(X_i)}{\pi_\beta(X_i)} - \frac{(1 - T_i)\pi'_\beta(X_i)}{1 - \pi_\beta(X_i)} \right\} = 0
\]
Weighting to Balance Covariates

Balancing condition: \( E \left\{ \frac{T_i X_i}{\pi_\beta(X_i)} - \frac{(1-T_i)X_i}{1-\pi_\beta(X_i)} \right\} = 0 \)
Generalized Method of Moments (GMM) Framework

- Just-identified CBPS: covariate balancing conditions alone
- Over-identified CBPS: combine them with score conditions

GMM (Hansen 1982):

$$\hat{\beta}_{\text{GMM}} = \arg\min_{\beta \in \Theta} \bar{g}_\beta(T, X)^\top \Sigma_\beta(T, X)^{-1} \bar{g}_\beta(T, X)$$

where

$$\bar{g}_\beta(T, X) = \frac{1}{N} \sum_{i=1}^{N} \begin{pmatrix} \text{score condition} \\ \text{balancing condition} \end{pmatrix}$$

- “Continuous updating” GMM estimator for $$\Sigma$$

Kosuke Imai (Princeton)
Covariate Balancing Propensity Score
Laval/Montreal (October 2014)
### CBPS Makes Weighting Methods Work Better

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CBPS Sacrifices Likelihood for Better Balance

Log-Likelihood

Covariate Imbalance

Likelihood-Balance Tradeoff

Both Models Specified Correctly

Neither Model Specified Correctly

Kosuke Imai (Princeton)  Covariate Balancing Propensity Score  Laval/Montreal (October 2014)
Revisiting Smith and Todd (2005)

- Evaluation bias: “true” bias = 0
- CBPS improves propensity score matching across specifications and matching methods
- However, specification test rejects the null

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<th>Optimal 1-to-N Nearest Neighbor</th>
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<td>(1256.84)</td>
<td>(1229.63)</td>
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LaLonde, Dehejia and Wahba, and others did this comparison.

Experimental estimate: $866 \text{ (s.e. } = 488)\text{.}

LaLonde + PSID pose a challenge: e.g., GenMatch −571 (1108)
## upload the package
library("CBPS")

## load the LaLonde data
data(LaLonde)

## Estimate ATT weights via CBPS
fit <- CBPS(treat ~ age + educ + re75 + re74 +
            I(re75==0) + I(re74==0),
            data = LaLonde, ATT = TRUE)

summary(fit)

## matching via MatchIt
library(MatchIt)

## one to one nearest neighbor with replacement
m.out <- matchit(treat ~ 1, distance = fitted(fit),
                 method = "nearest", data = LaLonde,
                 replace = TRUE)

summary(m.out)
Extensions to Other Causal Inference Settings

- Propensity score methods are widely applicable
- Thus, CBPS is also widely applicable

Extensions of propensity score to general treatment regimes
- Weighting (e.g., Imbens, 2000; Robins et al., 2000)
- Subclassification (e.g., Imai & van Dyk, 2004)
- Regression (e.g., Hirano & Imbens, 2004)

But, propensity score is mostly applied to binary treatment
- All existing methods assume correctly estimated propensity score
- No reliable methods to estimate generalized propensity score
- Harder to check balance across a non-binary treatment
- Many researchers dichotomize the treatment

Estimate the generalized propensity score such that covariate is balanced across all treatment groups
Two Motivating Examples

1. **Effect of education on political participation**
   - Education is assumed to play a key role in political participation.
   - \( T_i \): 3 education levels (graduated from college, attended college but not graduated, no college).
   - Original analysis \( \rightarrow \) **dichotomization** (some college vs. no college).
   - Propensity score matching.
   - Critics employ different matching methods.

2. **Effect of advertisements on campaign contributions**
   - Do TV advertisements increase campaign contributions?
   - \( T_i \): Number of advertisements aired in each zip code.
   - ranges from 0 to 22,379 advertisements.
   - Original analysis \( \rightarrow \) **dichotomization** (over 1000 vs. less than 1000).
   - Propensity score matching followed by linear regression with an original treatment variable.
Covariates are Not Balanced for Original Treatment

Kam and Palmer

Original vs. Genetic Matching

Graduated vs. Some College

Graduated vs. No College

Some vs. No College

Absolute Difference in Standardized Means

0.0 0.2 0.4 0.6 0.8 1.0

Kosuke Imai (Princeton)
Covariates are Not Balanced for Original Treatment

Urban and Niebler

Main Variables

Fixed Effects

Absolute Pearson Correlations

Original

○ Propensity Score Matching
Consider a 3 treatment value case as in our motivating example.

Generalized propensity score:
1. \( \pi_1^1(X_i) = \Pr(Y_i = 1 \mid X_i) \)
2. \( \pi_1^2(X_i) = \Pr(Y_i = 2 \mid X_i) \)

Standard estimation: multinomial logit regression

Sample balance conditions with orthogonalized contrasts:

\[
\bar{g}_\beta(T, X) = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{1\{T_i=1\}}{\pi_1^1(X_i)} - \frac{1\{T_i=2\}}{\pi_1^2(X_i)} + 2 \frac{1\{T_i=0\}}{1-\pi_1^1(X_i) - \pi_1^2(X_i)} \right) X_i
\]

GMM estimation as before
CBPS for a Continuous Treatment

- Generalized propensity score: \( f(T_i \mid X_i) \)
- Standard model: linear regression
- The stabilized weights (Robins et al.):
  \[
  \frac{f(T_i)}{f(T_i \mid X_i)}
  \]

- Covariate balancing condition:
  \[
  \mathbb{E}\left( \frac{f(T_i^*)}{f(T_i^* \mid X_i^*)} T_i^* X_i^* \right) = \int \left\{ \int \frac{f(T_i^*)}{f(T_i^* \mid X_i^*)} T_i^* dF(T_i^* \mid X_i^*) \right\} X_i^* dF(X_i^*)
  = \mathbb{E}(T_i^*) \mathbb{E}(X_i^*) = 0.
  \]
  where \( T_i^* \) and \( X_i^* \) are centered versions of \( T_i \) and \( X_i \)

- Again, estimate the generalized propensity score such that covariate balance is optimized
CBPS achieves better covariate balance
CBPS Avoids Extremely Large Weights

Kosuke Imai (Princeton)  Covariate Balancing Propensity Score  Laval/Montreal (October 2014)
CBPS Balances Well for a Dichotomized Treatment

Propensity Score Matching (Kam and Palmer)

Genetic Matching (Henderson and Chatfield)

ML Propensity Score Weighting
Empirical Results: Graduation Matters, Efficiency Gain

Effect on Political Participation

Some College Graduated Dichotomized

ML
CBPS
ML
CBPS
ML
CBPS

Kosuke Imai (Princeton) Covariate Balancing Propensity Score Laval/Montreal (October 2014)
Onto the Advertisement Example

![Box plot diagram showing Absolute Pearson Correlations for CBPS, ML, and Original for Main Variables and Fixed Effects.]

- **Original**
  - Box plot
- **ML**
  - Box plot
- **CBPS**
  - Box plot

- **Main Variables**
  - Fixed Effects

Kosuke Imai (Princeton)  
Covariate Balancing Propensity Score  
Laval/Montreal (October 2014)
Empirical Finding: Some Effect of Advertisement
Concluding Remarks

- Numerous advances in generalizing propensity score methods to non-binary treatments
- Yet, many applied researchers don’t use these methods and dichotomize non-binary treatments
- We offer a simple method to improve the estimation of propensity score for general treatment regimes
- Open-source R package: CBPS: Covariate Balancing Propensity Score available at CRAN

- Ongoing extensions:
  1. nonparametric estimation via empirical likelihood
  2. generalizing instrumental variables estimates
  3. spatial treatments