When Should We Use Linear Fixed Effects Regression Models for Causal Inference with Longitudinal Data?

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Fixed Effects Regressions in Causal Inference

- Linear fixed effects regression models are the primary workhorse for causal inference with longitudinal/panel data
- Researchers use them to adjust for unobserved time-invariant confounders (omitted variables, endogeneity, selection bias, ...):
  - "Good instruments are hard to find ..., so we'd like to have other tools to deal with unobserved confounders. This chapter considers ... strategies that use data with a time or cohort dimension to control for unobserved but fixed omitted variables" (Angrist & Pischke, *Mostly Harmless Econometrics*)
  - "fixed effects regression can scarcely be faulted for being the bearer of bad tidings" (Green *et al.*, *Dirty Pool*)
- When should we use linear FE regression models for causal inference?

Overview of the Talk

1. Identification: two under-appreciated causal assumptions of unit fixed effects regression estimators

2. Estimation: new matching framework for causal inference with panel data

3. Extend the analysis to two-way fixed effects models, difference-in-differences design, and synthetic control method

4. An empirical illustration: Effects of GATT on trade

Linear Regression with Unit Fixed Effects

- Balanced panel data with N units and T time periods
- Y_{it}: outcome variable
- X_{it}: binary causal or treatment variable of interest

Assumption 1 (Linearity)

$$Y_{it} = \alpha_i + \beta X_{it} + \epsilon_{it}$$

- $\mathbf{U}_i$: a vector of unobserved time-invariant confounders
- $\alpha_i = h(\mathbf{U}_i)$ for *any* function $h(\cdot)$
- A flexible way to adjust for unobservables
- Average contemporaneous treatment effect:

$$\beta = \mathbb{E}(Y_{it}(1) - Y_{it}(0))$$

Strict Exogeneity and Least Squares Estimator

Assumption 2 (Strict Exogeneity)

$$\epsilon_{it} \perp\!\!\!\perp \{\mathbf{X}_i, \mathbf{U}_i\}$$

- Mean independence is sufficient: $\mathbb{E}(\epsilon_{it} \mid \mathbf{X}_i, \mathbf{U}_i) = \mathbb{E}(\epsilon_{it}) = 0$
- Least squares estimator based on de-meaning:

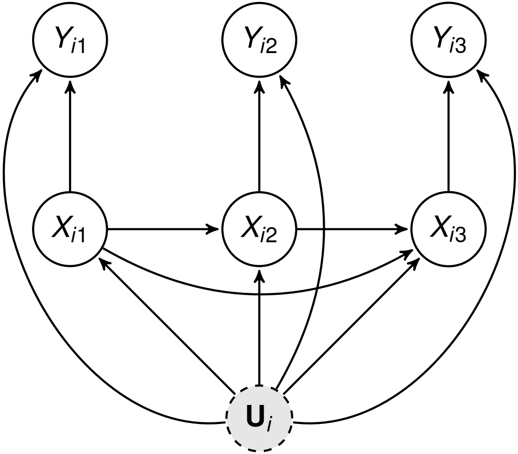
$$\hat{\beta}_{\text{FE}} = \underset{\beta}{\arg\min} \sum_{i=1}^{N} \sum_{t=1}^{T} \{(Y_{it} - \overline{Y}_i) - \beta(X_{it} - \overline{X}_i)\}^2$$

  where $\overline{X}_i$ and $\overline{Y}_i$ are unit-specific sample means
- ATE among those units with variation in treatment:

$$\tau = \mathbb{E}(Y_{it}(1) - Y_{it}(0) \mid C_{it} = 1)$$

  where $C_{it} = \mathbf{1}\{0 < \sum_{t=1}^{T} X_{it} < T\}$.

Causal Directed Acyclic Graph (DAG)



- arrow $=$ direct causal effect
- absence of arrows $\rightsquigarrow$ causal assumptions

Nonparametric Structural Equation Model (NPSEM)

- One-to-one correspondence with a DAG:

$$
\begin{aligned}
Y_{it} &= g_1(X_{it}, \mathbf{U}_i, \epsilon_{it}) \\
X_{it} &= g_2(X_{i1}, \ldots, X_{i,t-1}, \mathbf{U}_i, \eta_{it})
\end{aligned}
$$

- Nonparametric generalization of linear unit fixed effects model:
  - Allows for nonlinear relationships, effect heterogeneity
  - Strict exogeneity holds ($\epsilon_{it} \to Y_{it} \leftarrow \{\mathbf{X}_i, \mathbf{U}_i\}$)
  - No arrows can be added without violating Assumptions 1 and 2
- Causal assumptions:
  1. No unobserved time-varying confounders
  2. Past outcomes do not directly affect current outcome
  3. Past outcomes do not directly affect current treatment
  4. Past treatments do not directly affect current outcome

Potential Outcomes Framework

- DAG $\rightsquigarrow$ causal structure
- Potential outcomes $\rightsquigarrow$ treatment assignment mechanism

Assumption 3 (No carryover effect)

Past treatments do not directly affect current outcome

$$Y_{it}(X_{i1}, X_{i2}, \ldots, X_{i,t-1}, X_{it}) = Y_{it}(X_{it})$$

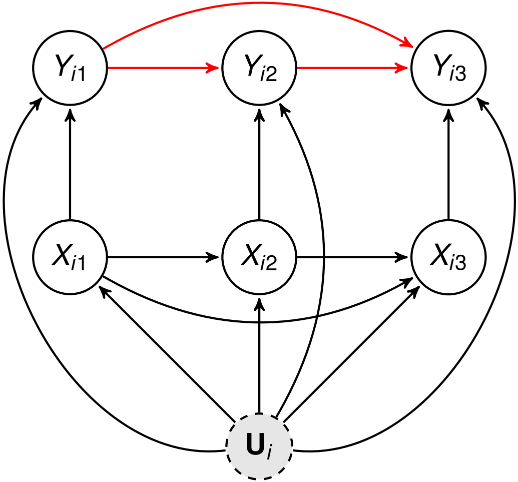
- What randomized experiment satisfies unit fixed effects model?
  1. randomize X_{i1} given $\mathbf{U}_i$
  2. randomize X_{i2} given X_{i1} and $\mathbf{U}_i$
  3. randomize X_{i3} given X_{i2}, X_{i1}, and $\mathbf{U}_i$
  4. and so on

Assumption 4 (Sequential Ignorability with Unobservables)

$$\{Y_{it}(1), Y_{it}(0)\}_{t=1}^{T} \perp\!\!\!\perp X_{i1} \mid \mathbf{U}_i$$
$$\vdots$$
$$\{Y_{it}(1), Y_{it}(0)\}_{t=1}^{T} \perp\!\!\!\perp X_{it'} \mid X_{i1}, \ldots, X_{i,t'-1}, \mathbf{U}_i$$
$$\vdots$$
$$\{Y_{it}(1), Y_{it}(0)\}_{t=1}^{T} \perp\!\!\!\perp X_{iT} \mid X_{i1}, \ldots, X_{i,T-1}, \mathbf{U}_i$$

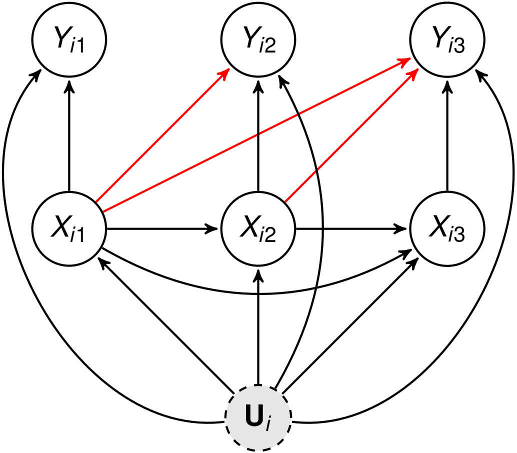
- "as-if random" assumption without conditioning on past outcomes
- Past outcomes cannot directly affect current treatment
- Says nothing about whether past outcomes can directly affect current outcome

Past Outcomes Directly Affect Current Outcome



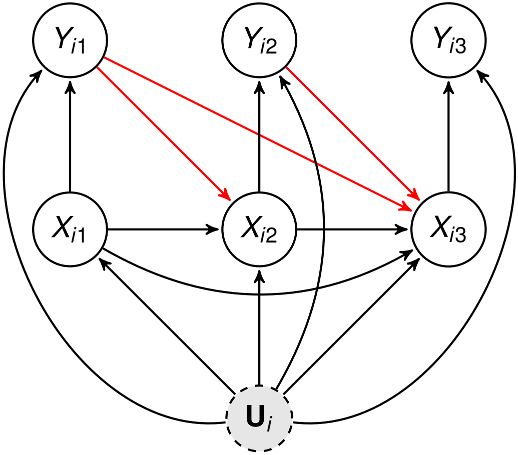
- Strict exogeneity still holds
- Past outcomes do not confound $X_{it} \longrightarrow Y_{it}$ given $\mathbf{U}_i$
- No need to adjust for past outcomes

Past Treatments Directly Affect Current Outcome



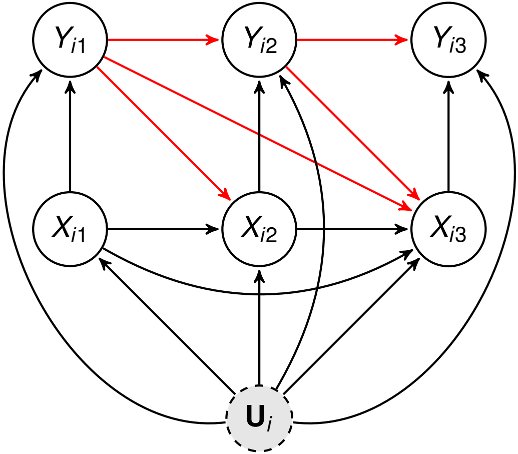
- Past treatments as confounders
- Need to adjust for past treatments
- Strict exogeneity holds given past treatments and $\mathbf{U}_i$
- Impossible to adjust for an entire treatment history and $\mathbf{U}_i$ at the same time
- Adjust for a small number of past treatments $\rightsquigarrow$ often arbitrary

Past Outcomes Directly Affect Current Treatment



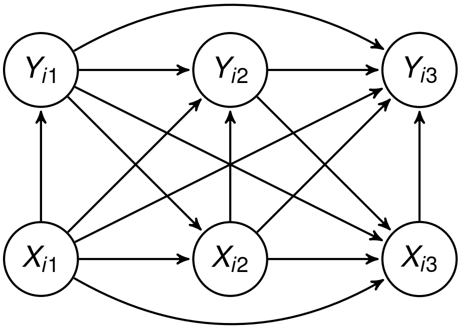
- Correlation between error term and future treatments
- Violation of strict exogeneity
- No adjustment is sufficient
- Together with the previous assumption $\rightsquigarrow$ no feedback effect over time

Instrumental Variables Approach



- Instruments: X_{i1}, X_{i2}, and Y_{i1}
- GMM: Arellano and Bond (1991)
- Exclusion restrictions
- Arbitrary choice of instruments
- Substantive justification rarely given

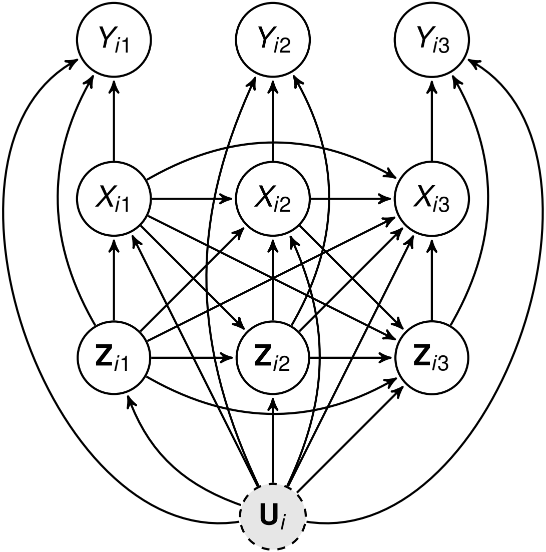
An Alternative Selection-on-Observables Approach



- Absence of unobserved time-invariant confounders $\mathbf{U}_i$
- past treatments can directly affect current outcome
- past outcomes can directly affect current treatment

- Comparison across units within the same time rather than across different time periods within the same unit
- Marginal structural models $\rightsquigarrow$ can identify the average effect of an entire treatment sequence
- Trade-off $\rightsquigarrow$ no free lunch

Adjusting for Observed Time-varying Confounders



- past treatments cannot directly affect current outcome
- past outcomes cannot directly affect current treatment
- adjusting for $\mathbf{Z}_{it}$ does not relax these assumptions
- past outcomes cannot *indirectly* affect current treatment through $\mathbf{Z}_{it}$

Linear Fixed Effects Estimator

- Even if these assumptions are satisfied, the the unit fixed effects estimator is inconsistent for the ATE:

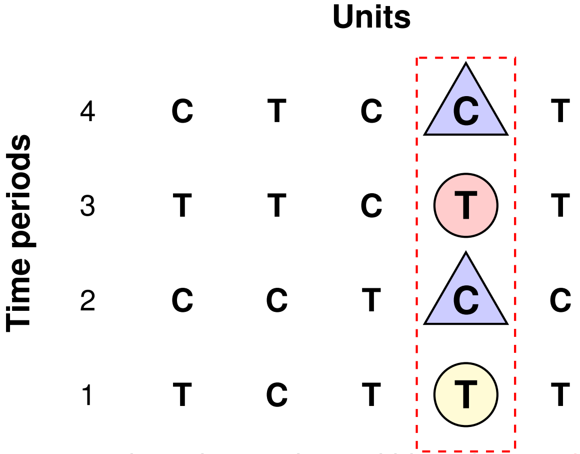
$$\hat{\beta}_{\text{FE}} \xrightarrow{p} \frac{\mathbb{E}\left\{C_i\left(\frac{\sum_{t=1}^{T} X_{it}Y_{it}}{\sum_{t=1}^{T} X_{it}} - \frac{\sum_{t=1}^{T}(1-X_{it})Y_{it}}{\sum_{t=1}^{T} 1-X_{it}}\right)S_i^2\right\}}{\mathbb{E}(C_i S_i^2)} \neq \tau$$

  where $S_i^2 = \sum_{t=1}^{T}(X_{it} - \overline{X}_i)^2/(T-1)$ is the unit-specific variance

- Can we improve the unit fixed effects estimator?

A Matching Framework

- Unit specific confounder: Matching observations within unit



- FE estimator matches observations within unit regardless of the treatment status (de-meaning) $\rightsquigarrow$ model-based adjustment

The Within-Unit Matching Estimator

- Key idea: comparison across time periods within the same unit
- For the within-unit matching estimator,

$$\mathcal{M}_{it}^{\text{match}} = \{(i', t') : i' = i, X_{i't'} = 1 - X_{it}\}$$

- $\mathcal{M}_{it}$: matched set for observation (i, t)
- A general matching estimator:

$$\hat{\tau}_{\text{match}} = \frac{1}{\sum_{i=1}^{N}\sum_{t=1}^{T} D_{it}} \sum_{i=1}^{N}\sum_{t=1}^{T} D_{it}\left(\widehat{Y_{it}(1)} - \widehat{Y_{it}(0)}\right)$$

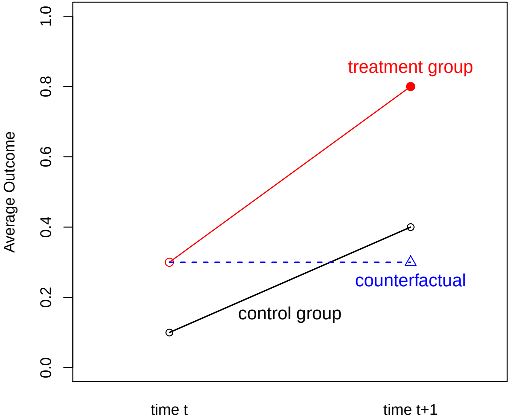
where $D_{it} = \mathbf{1}\{\#\mathcal{M}_{it} > 0\}$ and

$$\widehat{Y_{it}(x)} = \begin{cases} Y_{it} & \text{if } X_{it} = x \\ \frac{1}{\#\mathcal{M}_{it}} \sum_{(i',t') \in \mathcal{M}_{it}} Y_{i't'} & \text{if } X_{it} = 1 - x \end{cases}$$

Before-and-After Design

- Accommodating various causal quantity of interest
- No time trend for the average potential outcomes:

$$\mathbb{E}(Y_{it}(x) - Y_{i,t-1}(x) \mid X_{it} \neq X_{i,t-1}) \quad = \quad 0 \quad \text{for} \quad x = 0, 1$$



- This is a matching estimator with the following matched set:

$$\mathcal{M}(i,t) = \{(i',t') : i' = i, t' \in \{t-1, t+1\}, X_{i't'} = 1 - X_{it}\}$$

- It is also the first differencing estimator:

$$\hat{\beta}_{\text{FD}} = \underset{\beta}{\arg\min} \sum_{i=1}^{N} \sum_{t=2}^{T} \{(Y_{it} - Y_{i,t-1}) - \beta(X_{it} - X_{i,t-1})\}^2$$

- "We emphasize that the model and the interpretation of β are *exactly* as in [the linear fixed effects model]. What differs is our method for estimating β" (Wooldridge; italics original).
- The identification assumptions is very different
- But, still requires the assumption that past outcomes do not affect current treatment (Regression towards the mean)

Matching as a Weighted Unit Fixed Effects Estimator

- *Any* within-unit matching estimator can be written as a weighted unit fixed effects estimator with different regression weights
- The proposed within-matching estimator:

$$\hat{\tau}_{\text{match}} = \hat{\beta}_{\text{WFE}} = \underset{\beta}{\arg\min} \sum_{i=1}^{N} \sum_{t=1}^{T} D_{it} W_{it} \{(Y_{it} - \overline{Y}_i^*) - \beta(X_{it} - \overline{X}_i^*)\}^2$$

where $\overline{X}_i^*$ and $\overline{Y}_i^*$ are unit-specific weighted averages

- Example based on $\mathcal{M}_{it}^{\text{match}}$:

$$W_{it} = \begin{cases} \frac{T}{\sum_{t'=1}^{T} X_{it'}} & \text{if} \quad X_{it} = 1, \\ \frac{T}{\sum_{t'=1}^{T} (1 - X_{it'})} & \text{if} \quad X_{it} = 0. \end{cases}$$

- We show how to construct regression weights for different matching estimators (i.e., different matched sets)
- Idea: count the number of times each observation is used for matching

- Benefits:
  - computational efficiency
  - model-based standard errors
  - robustness $\rightsquigarrow$ matching estimator is consistent even when linear unit fixed effects regression is the true model
  - specification test (White 1980) $\rightsquigarrow$ null hypothesis: linear fixed effects regression is the true model

Linear Regression with Unit and Time Fixed Effects

- Model:

$$Y_{it} = \alpha_i + \gamma_t + \beta X_{it} + \epsilon_{it}$$

where γ_t flexibly adjusts for a vector of unobserved unit-invariant time effects $\mathbf{V}_t$, i.e., $\gamma_t = f(\mathbf{V}_t)$

- Estimator:

$$\hat{\beta}_{\text{FE2}} = \arg\min_{\beta} \sum_{i=1}^{N} \sum_{t=1}^{T} \{(Y_{it} - \overline{Y}_i - \overline{Y}_t + \overline{Y}) - \beta(X_{it} - \overline{X}_i - \overline{X}_t + \overline{X})\}^2$$

where $\overline{Y}_t$ and $\overline{X}_t$ are time-specific means, and $\overline{Y}$ and $\overline{X}$ are overall means

Understanding the Two-way Fixed Effects Estimator

The two-way FE estimator combines three biased estimators!

1. β_{FE}: bias due to time effects
2. β_{FEtime}: bias due to unit effects
3. β_{pool}: bias due to both time and unit effects

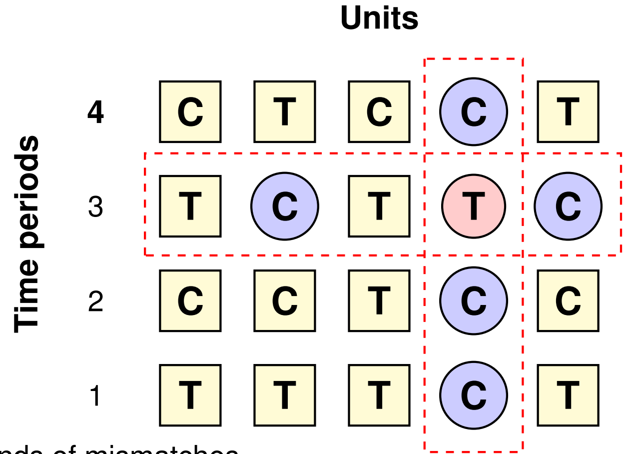
$$\hat{\beta}_{\text{FE2}} = \frac{\omega_{\text{FE}} \times \hat{\beta}_{\text{FE}} + \omega_{\text{FEtime}} \times \hat{\beta}_{\text{FEtime}} - \omega_{\text{pool}} \times \hat{\beta}_{\text{pool}}}{w_{\text{FE}} + w_{\text{FEtime}} - w_{\text{pool}}}$$

with sufficiently large N and T, the weights are given by,

$$\begin{aligned} \omega_{\text{FE}} &\approx \mathbb{E}(S_i^2) = \text{average unit-specific variance} \\ \omega_{\text{FEtime}} &\approx \mathbb{E}(S_t^2) = \text{average time-specific variance} \\ \omega_{\text{pool}} &\approx S^2 = \text{overall variance} \end{aligned}$$

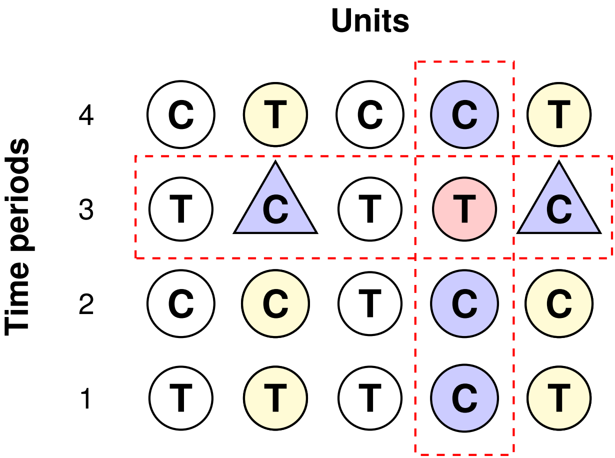
Matching and Two-way Fixed Effects Estimators

- Problem: No other unit shares the same unit and time



- Two kinds of mismatches
  1. Same treatment status
  2. Neither same unit nor same time

We Can Never Eliminate Mismatches

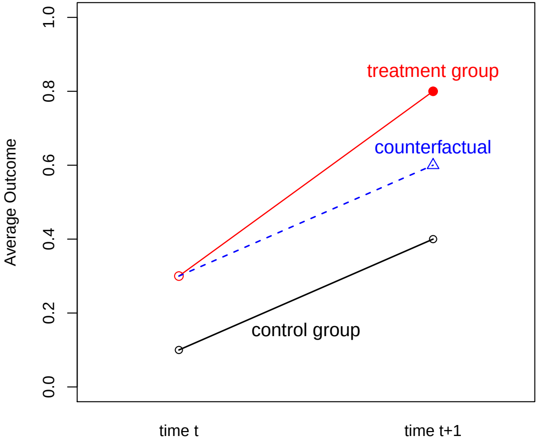


- To cancel time and unit effects, we must induce mismatches
- Solution: Difference-in-Differences

Difference-in-Differences Design

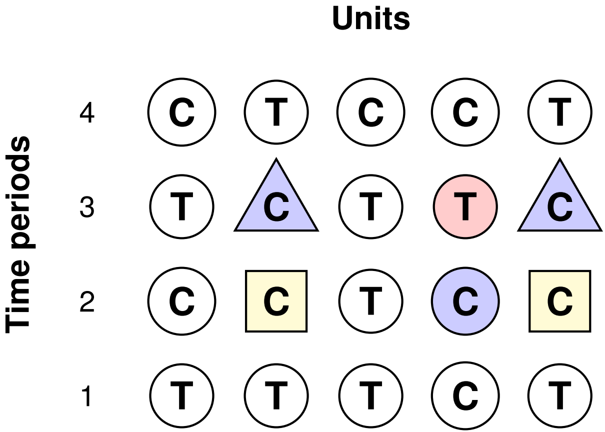
- Parallel trend assumption:

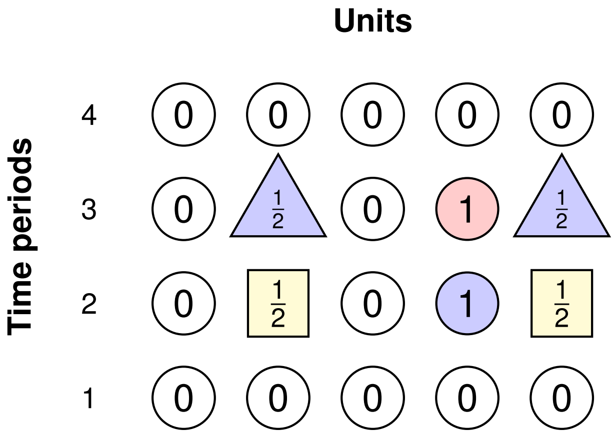
$$
\begin{aligned}
& \mathbb{E}(Y_{it}(0) - Y_{i,t-1}(0) \mid X_{it} = 1, X_{i,t-1} = 0) \\
= \; & \mathbb{E}(Y_{it}(0) - Y_{i,t-1}(0) \mid X_{it} = X_{i,t-1} = 0)
\end{aligned}
$$



General DiD $=$ Weighted Two-Way FE Effects

- 2×2: equivalent to linear two-way fixed effects regression
- General setting: Multiple time periods, repeated treatments





- Fast computation, standard error, specification test
- Still assumes that past outcomes don't affect current treatment
- Baseline outcome difference $\rightsquigarrow$ caused by unobserved time-invariant confounders
- It should not reflect causal effect of baseline outcome on treatment assignment

Synthetic Control Method (Abadie et al. 2010)

- One treated unit i^* receiving the treatment at time T
- Quantity of interest: $Y_{i^*T} - Y_{i^*T}(0)$
- Create a synthetic control using past outcomes
- Weighted average: $\widehat{Y_{i^*T}(0)} = \sum_{i \neq i^*} \hat{w}_i Y_{iT}$
- Estimate weights to balance past outcomes and past time-varying covariates

- A motivating autoregressive model:

$$\begin{aligned} Y_{iT}(0) &= \rho_T Y_{i,T-1}(0) + \delta_T^\top \mathbf{Z}_{iT} + \epsilon_{iT} \\ \mathbf{Z}_{iT} &= \lambda_{T-1} Y_{i,T-1}(0) + \Delta_T \mathbf{Z}_{i,T-1} + \nu_{iT} \end{aligned}$$

- Past outcomes can affect current treatment
- No unobserved time-invariant confounders

Causal Effect of ETA's Terrorism

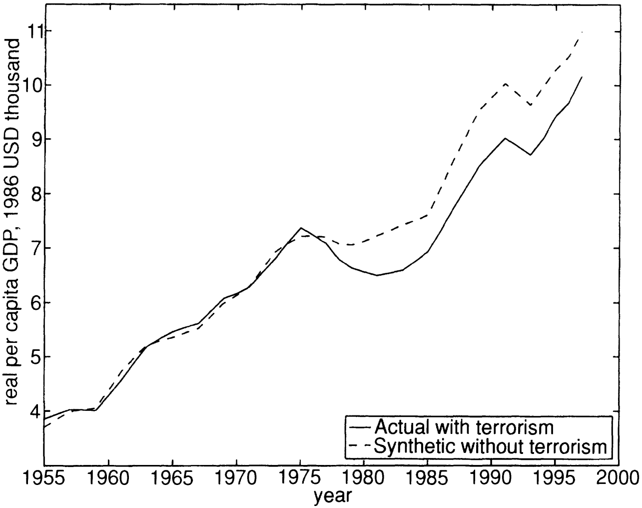


FIGURE 1. PER CAPITA GDP FOR THE BASQUE COUNTRY

Abadie and Gardeazabal (2003, AER)

- The main motivating model:

$$Y_{it}(0) \quad = \quad \gamma_t + \delta_t^\top \mathbf{Z}_{it} + \xi^\top \mathbf{U}_i + \epsilon_{it}$$

- A generalization of the linear two-way fixed effects model
- How is it possible to adjust for unobserved time-invariant confounders by adjusting for past outcomes?

- The key assumption: there exist weights such that

$$\sum_{i \neq i^*} w_i \mathbf{Z}_{it} = \mathbf{Z}_{i^* t} \text{ for all } t \leq T-1 \quad \text{and} \quad \sum_{i \neq i^*} w_i \mathbf{U}_i = \mathbf{U}_{i^*}$$

- In general, adjusting for observed confounders does not adjust for unobserved confounders

- The same tradeoff as before

Effects of GATT Membership on International Trade

1. Controversy
   - Rose (2004): No effect of GATT membership on trade
   - Tomz et al. (2007): Significant effect with non-member participants
2. The central role of fixed effects models:
   - Rose (2004): one-way (year) fixed effects for dyadic data
   - Tomz *et al.* (2007): two-way (year and dyad) fixed effects
   - Rose (2005): "I follow the profession in placing most confidence in the fixed effects estimators; I have no clear ranking between country-specific and country pair-specific effects."
   - Tomz *et al.* (2007): "We, too, prefer FE estimates over OLS on both theoretical and statistical ground"

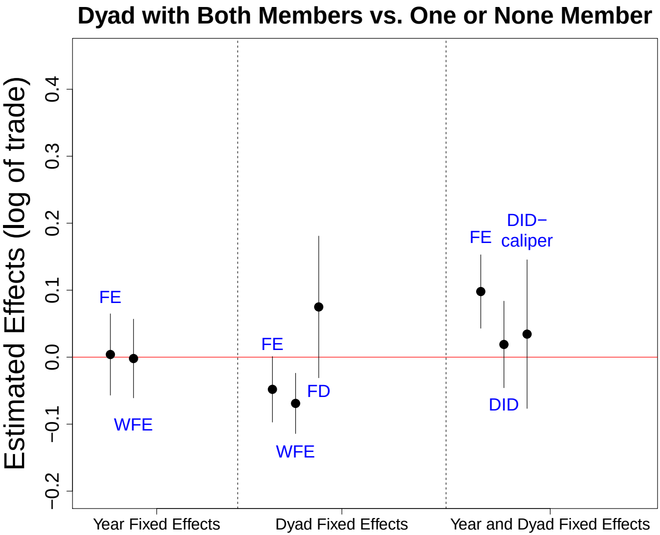
Data and Methods

1. Data
   - Data set from Tomz et al. (2007)
   - Effect of GATT: 1948 – 1994
   - 162 countries, and 196,207 (dyad-year) observations
2. Year fixed effects model:

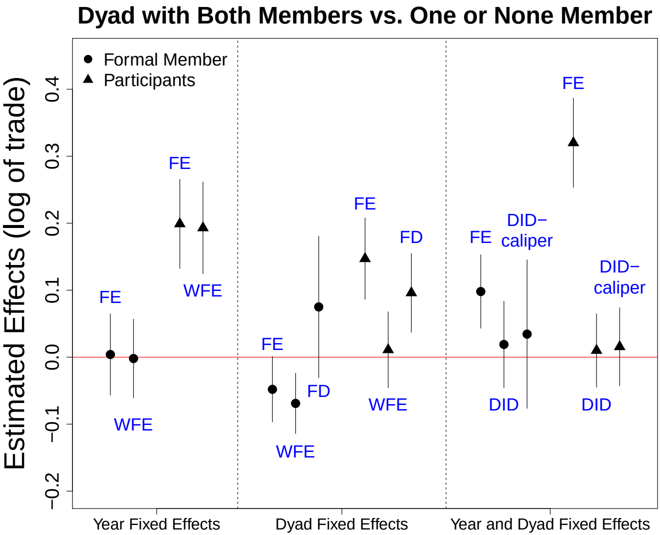
$$\ln Y_{it} = \alpha_t + \beta X_{it} + \delta^\top \mathbf{Z}_{it} + \epsilon_{it}$$

   - Y_{it}: trade volume
   - X_{it}: membership (formal/participants) Both vs. At most one
   - $\mathbf{Z}_{it}$: 15 dyad-varying covariates (e.g., log product GDP)
3. Assumptions:
   - past membership status doesn't directly affect current trade volume
   - past trade volume doesn't affect current membership status
   - Before-and-after $\rightsquigarrow$ increasing trend in trade volume
   - Difference-in-differences after conditional on past outcome?

Empirical Results: Formal Membership



Empirical Results: Participants Included



Concluding Remarks

- When should we use linear fixed effects models?
- Key tradeoff:
  1. unobserved time-invariant confounders $\rightsquigarrow$ fixed effects
  2. causal dynamics between treatment and outcome $\rightsquigarrow$ selection-on-observables
- Two key (under-appreciated) causal assumptions of fixed effects:
  1. past treatments do not directly affect current outcome
  2. past outcomes do not directly affect current treatment
- A new matching estimator:
  1. Within-unit matching estimator $\rightsquigarrow$ no linearity assumption
  2. Various causal identification strategies can be incorporated including the before-and-after and difference-in-differences designs
  3. Equivalent representation as a weighted linear fixed effects regression estimator
- R package **wfe** is available at CRAN

Send comments and suggestions to:

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More information about this and other research:

http://imai.princeton.edu
http://web.mit.edu/insong/www

Extra Slides

- D-deperation

- Mismatches in FE
- Treat vs. Weights

- Time-varying Confounder
- Matching on Z_{it}

D-separation (Pearl, 1988)

Consider a directed graph in which A, B, and C are arbitrary nonintersecting sets of nodes.

Definition 1 (A path from any node in A to any node in B is **blocked** if it includes a node such that either)

1. the arrows on the path meet either head-to-tail or tail-to-tail at the node, and the node is in the set C, or
2. the arrows meet head-to-head (collider) at the node, and neither the node, nor any of its descendants, is in the set C

If all paths are blocked, then A is said to be d-separated from B by C implying conditional independence

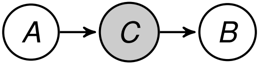


Figure: (a) head-to-tail

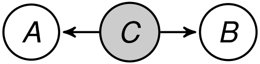


Figure: (b) tail-to-tail

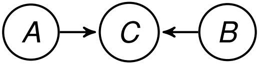


Figure: (c) head-to-head

Figure: Conditional Independence $\mathbf{A \perp\!\!\!\perp B \mid C}$ implied by a DAG

Unit Fixed Effects Estimator as a Matching Estimator

- "de-meaning" $\rightsquigarrow$ match with all other observations within the same unit:

$$\mathcal{M}(i,t) \quad = \quad \{(i',t') : i' = i, t' \neq t\}$$

- mismatch: observations with the same treatment status
- Unit fixed effects estimator adjusts for mismatches:

$$\hat{\beta}_{\text{FE}} \quad = \quad \frac{1}{K}\left\{\frac{1}{\sum_{i=1}^{N}\sum_{t=1}^{T} D_{it}}\sum_{i=1}^{N}\sum_{t=1}^{T} D_{it}\left(\widehat{Y_{it}(1)} - \widehat{Y_{it}(0)}\right)\right\}$$

where K is the proportion of proper matches

- The within-unit matching estimator eliminates all mismatches

Adjusting for Time-varying Confounders

- $\mathbf{Z}_{it}$: observed time-varying confounders
- Model:

$$Y_{it} \quad = \quad \alpha_i + \beta X_{it} + \gamma^\top \mathbf{Z}_{it} + \epsilon_{it}$$

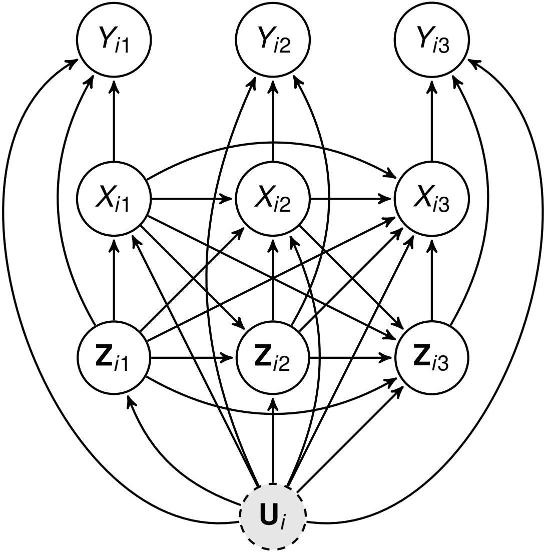
- No carryover effect assumption

Assumption 5 (Strict Exogeneity with Time-varying Confounders)

$$\mathbb{E}(\epsilon_{it} \mid \mathbf{X}_i, \mathbf{Z}_i, \mathbf{U}_i) \ = \ 0$$

where $\mathbf{Z}_i$ is the $T \times 1$ vector of time-varying confounders

Adjusting for Observed Time-varying Confounders



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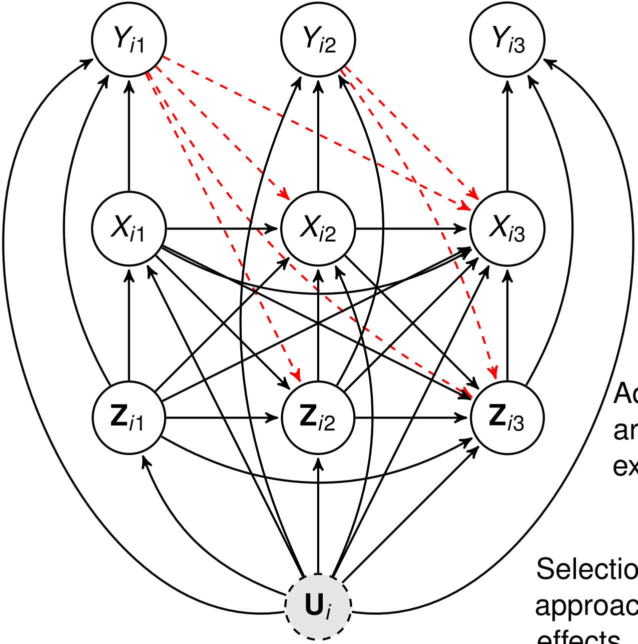
- adjusting for $\mathbf{Z}_{it}$ does not relax these assumptions
- past outcomes cannot *indirectly* affect current treatment through $\mathbf{Z}_{it}$



Adding a red dashed arrow violates strict exogeneity

Selection-on-observables approach allows for these effects

Covariate-adjusted Within-Unit Matching Estimator

- Example: within-unit nearest neighbor matching

$$\mathcal{M}(i,t) \quad = \quad \{(i',t') : i' = i, X_{i't'} = 1 - X_{it}, \mathcal{D}(\mathbf{Z}_{it}, \mathbf{Z}_{i't'}) = J_{it}\}$$

  where $\mathcal{D}(\cdot,\cdot)$ is a distance measure (e.g., Mahalanobis distance),

$$J_{it} \quad = \quad \min_{(i,t') \in \mathcal{M}(i,t)} \mathcal{D}(\mathbf{Z}_{it}, \mathbf{Z}_{it'})$$

- This matching estimator can also be written as a weighted unit fixed effects estimator

Within-Unit Matching with Model-based Adjustment

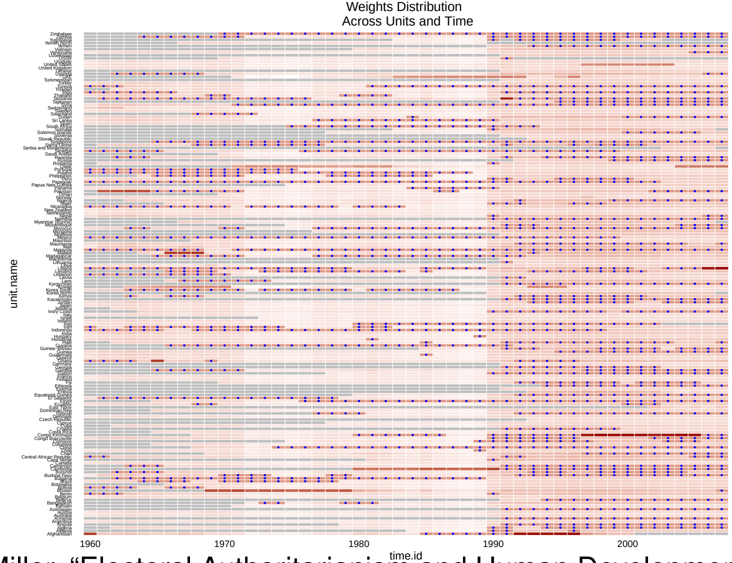
- An alternative way to adjust for time-varying covariates
- Weighted fixed effects estimator with time-varying covariates:

$$
\begin{aligned}
& (\hat{\beta}_{\text{WFEadj}}, \hat{\delta}_{\text{WFEadj}}) \\
= \quad & \underset{\beta,\delta}{\arg\min} \sum_{i=1}^{N} \sum_{t=1}^{T} W_{it} \{ (Y_{it} - \overline{Y}_i^*) - \beta (X_{it} - \overline{X}_i^*) - \delta^\top (\mathbf{Z}_{it} - \overline{\mathbf{Z}}_i^*) \}^2
\end{aligned}
$$

- This is a matching estimator:

$$
\widehat{Y_{it}(x)} = \begin{cases} Y_{it} - \hat{\delta}_{\text{WFEadj}}^\top \mathbf{Z}_{it} & \text{if } X_{it} = x \\ \frac{1}{\#\mathcal{M}(i,t)} \sum_{(i',t') \in \mathcal{M}(i,t)} Y_{i't'} - \hat{\delta}_{\text{WFEadj}}^\top \mathbf{Z}_{i't'} & \text{if } X_{it} = 1 - x \end{cases}
$$

Visualization of Treatment and Weights



Miller, “Electoral Authoritarianism and Human Development”