Causal Interaction in High Dimension

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Interaction Effects and Causal Heterogeneity

1. **Moderation**
   - How do treatment effects vary across individuals?
   - Who benefits from (or is harmed by) the treatment?
   - Interaction between treatment and pre-treatment covariates

2. **Causal interaction**
   - What aspects of a treatment are responsible for causal effects?
   - What combination of treatments is efficacious?
   - Interaction between treatment variables

3. **Individualized treatment regimes**
   - What combination of treatments is optimal for a given individual?
Causal Interaction in High Dimension

- High dimension = many treatments, each having multiple levels

- A motivating application: Conjoint analysis (Hainmueller et al. 2014)
  - survey experiments to measure immigration preferences
  - a representative sample of 1,396 American adults
  - each respondent evaluates 5 pairs of immigrant profiles
  - gender\(^2\), education\(^7\), origin\(^{10}\), experience\(^4\), plan\(^4\), language\(^4\), profession\(^{11}\), application reason\(^3\), prior trips\(^5\)
  - Over 1 million treatment combinations!
  - What combinations of immigrant characteristics make them preferred?

- Too many treatment combinations \(\Rightarrow\) Need for an effective summary
- Interaction effects play an essential role
Two Interpretations of Causal Interaction

1 **Conditional effect interpretation:**
   - Does the effect of one treatment change as we vary the value of another treatment?
   - Does the effect of being black change depending on whether an applicant is male or female?
   - Useful for testing moderation among treatments

2 **Interactive effect interpretation:**
   - Does a combination of treatments induce an *additional effect* beyond the sum of separate effects attributable to each treatment?
   - Does being a black female induce an additional effect beyond the effect of being black and that of being female?
   - Useful for finding efficacious treatment combinations in high dimension
An Illustration in the $2 \times 2$ Case

- Two binary treatments: $A$ and $B$
- Potential outcomes: $Y(a, b)$ where $a, b \in \{0, 1\}$
- Conditional effect interpretation:

$$\underbrace{[Y(1, 1) - Y(0, 1)]} - \underbrace{[Y(1, 0) - Y(0, 0)]}$$

$\Rightarrow$ requires the specification of moderator

- Interactive effect interpretation:

$$\underbrace{[Y(1, 1) - Y(0, 0)]} - \underbrace{[Y(1, 0) - Y(0, 0)]} - \underbrace{[Y(0, 1) - Y(0, 0)]}$$

$\Rightarrow$ requires the specification of baseline condition

- The same quantity but two different interpretations
Difficulty of the Conventional Approach

- **Lack of invariance** to the baseline condition
  \[ \Rightarrow \text{Inference depends on the choice of baseline condition} \]

- **3 \times 3** example:
  - Treatment \( A \in \{a_0, a_1, a_2\} \) and Treatment \( B \in \{b_0, b_1, b_2\} \)
  - Regression model with the baseline condition \((a_0, b_0)\):
    \[
    \mathbb{E}(Y \mid A, B) = 1 + a_1^* + a_2^* + b_2^* + a_1^*b_2^* + 2a_2^*b_2^* + 3a_2^*b_1^*
    \]
    - Interaction effect for \((a_2, b_2)\) > Interaction effect for \((a_1, b_2)\)
  - Another equivalent model with the baseline condition \((a_0, b_1)\):
    \[
    \mathbb{E}(Y \mid A, B) = 1 + a_1^* + 4a_2^* + b_2^* + a_1^*b_2^* - a_2^*b_2^* - 3a_2^*b_0^*
    \]
    - Interaction effect for \((a_2, b_2)\) < Interaction effect for \((a_1, b_2)\)
    - Interaction effect for \((a_2, b_1)\) is zero under the second model
    - All interaction effects with at least one baseline value are zero
Empirical Illustration: Lack of Invariance

- Linear regression with main effects and two-way interactions
- Baseline: \textit{lowest} levels of job experiences and education

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The Effects of Changing the Baseline Condition

- Same linear regression but different baseline
- Baseline: *highest* levels of job experiences and education

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(baseline)
The Contributions of the Paper

1 Problems of the conventional approach:
   - Lack of invariance to the choice of baseline condition
   - Difficulty of interpretation for higher-order interaction

2 Solution: Average Marginal Treatment Interaction Effect
   - invariant to baseline condition
   - same, intuitive interpretation even for high dimension
   - simple estimation procedure

3 Reanalysis of the immigration survey experiment
Two-way Causal Interaction

- Two factorial treatments:
  \[ A \in \mathcal{A} = \{a_0, a_1, \ldots, a_{D_A-1}\} \]
  \[ B \in \mathcal{B} = \{b_0, b_1, \ldots, b_{D_B-1}\} \]

- Assumption: **Full factorial design**
  1. Randomization of treatment assignment
     \[ \{ Y(a_\ell, b_m) \}_{a_\ell \in \mathcal{A}, b_m \in \mathcal{B}} \perp \perp \{A, B\} \]
  2. Non-zero probability for all treatment combination
     \[ \Pr(A = a_\ell, B = b_m) > 0 \quad \text{for all } a_\ell \in \mathcal{A} \quad \text{and} \quad b_m \in \mathcal{B} \]

- Fractional factorial design not allowed
  1. Use a small non-zero assignment probability
  2. Focus on a subsample
  3. Combine treatments
Non-Interaction Effects of Interest

1. **Average Treatment Combination Effect (ATCE):**
   - Average effect of treatment combination \((A, B) = (a_\ell, b_m)\) relative to the baseline condition \((A, B) = (a_0, b_0)\)
   
   \[ \tau(a_\ell, b_m; a_0, b_0) \equiv \mathbb{E}\{Y(a_\ell, b_m) - Y(a_0, b_0)\} \]
   
   - Which treatment combination is most efficacious?

2. **Average Marginal Treatment Effect (AMTE; Hainmueller et al. 2014):**
   - Average effect of treatment \(A = a_\ell\) relative to the baseline condition \(A = a_0\) averaging over the other treatment \(B\)
   
   \[ \psi(a_\ell, a_0) \equiv \int_B \mathbb{E}\{Y(a_\ell, B) - Y(a_0, B)\}dF(B) \]
   
   - Which treatment is effective on average?
The Conventional Approach to Causal Interaction

- **Average Treatment Interaction Effect (ATIE):**
  \[ \xi(a_\ell, b_m; a_0, b_0) \equiv \mathbb{E}\{Y(a_\ell, b_m) - Y(a_0, b_m) - Y(a_\ell, b_0) + Y(a_0, b_0)\} \]

- **Conditional effect interpretation:**
  \[ \mathbb{E}\{Y(a_\ell, b_m) - Y(a_0, b_m)\} - \mathbb{E}\{Y(a_\ell, b_0) - Y(a_0, b_0)\} \]
  Effect of \( A = a_\ell \) when \( B = b_m \)
  Effect of \( A = a_\ell \) when \( B = b_0 \)

- **Interactive effect interpretation:**
  \[ \tau(a_\ell, b_m; a_0, b_0) - \mathbb{E}\{Y(a_\ell, b_0) - Y(a_0, b_0)\} - \mathbb{E}\{Y(a_0, b_m) - Y(a_0, b_0)\} \]
  ATCE
  Effect of \( A = a_\ell \) when \( B = b_0 \)
  Effect of \( B = b_m \) when \( A = a_0 \)

- **Estimation:** Linear regression with interaction terms
Lack of Invariance to the Baseline Condition

- Comparison between two ATIEs should not be affected by the choice of baseline conditions.
- We prove that the ATIEs are *neither interval nor order invariant*.

**Interval invariance:**

\[
\xi(a_\ell, b_m; a_0, b_0) - \xi(a_\ell', b_m'; a_0, b_0) \\
= \xi(a_\ell, b_m; a_\tilde{\ell}, b_\tilde{m}) - \xi(a_\ell', b_m'; a_\tilde{\ell}, b_\tilde{m}),
\]

**Order invariance:**

\[
\xi(a_\ell, b_m; a_0, b_0) \geq \xi(a_\ell', b_m'; a_0, b_0) \\
\iff \xi(a_\ell, b_m; a_\tilde{\ell}, b_\tilde{m}) \geq \xi(a_\ell', b_m'; a_\tilde{\ell}, b_\tilde{m}).
\]
Ineffectiveness of Interaction Plot in High Dimension

Problem: it does not plot interaction effects themselves
ATIE is Sensitive to the Choice of Baseline Condition

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The New Causal Interaction Effect

- **Average Marginal Treatment Interaction Effect (AMTIE):**

\[ \pi(a_\ell, b_m; a_0, b_0) \equiv \tau(a_\ell, b_m; a_0, b_0) - \psi(a_\ell, a_0) - \psi(b_m, b_0) \]

\[ \text{ATCE of } (A, B) = (a_\ell, b_m) \quad \text{AMTE of } a_\ell \quad \text{AMTE of } b_m \]

- Interactive effect interpretation: additional effect induced by \( A = a_\ell \) and \( B = b_m \) together beyond the separate effect of \( A = a_\ell \) and that of \( B = b_m \)

- Compare this with ATIE:

\[ \tau(a_\ell, b_m; a_0, b_0) - \mathbb{E}\{Y(a_\ell, b_0) - Y(a_0, b_0)\} - \mathbb{E}\{Y(a_0, b_m) - Y(a_0, b_0)\} \]

\[ \text{ATCE} \quad \text{Effect of } A = a_\ell \text{ when } B = b_0 \quad \text{Effect of } B = b_m \text{ when } A = a_0 \]

- We prove that the AMTIEs are both *interval and order invariant*

- The AMTIEs do depend on the distribution of treatment assignment
  1. specified by one’s experimental design
  2. motivated by the target population
### AMTIE is Invariant to the Choice of Baseline Condition

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<tr>
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The Relationships between the ATIE and the AMTIE

1. The AMTIE is a linear function of the ATIEs:

\[ \pi(a_\ell, b_m; a_0, b_0) = \xi(a_\ell, b_m; a_0, b_0) - \sum_{a \in A} \Pr(A_i = a) \xi(a, b_m; a_0, b_0) \]

\[ - \sum_{b \in B} \Pr(B_i = b) \xi(a_\ell, b; a_0, b_0) \]

2. The ATIE is also a linear function of the AMTIEs:

\[ \xi(a_\ell, b_m; a_0, b_0) = \pi(a_\ell, b_m; a_0, b_0) - \pi(a_\ell, b_0; a_0, b_0) - \pi(a_0, b_m; a_0, b_0) \]

- Absence of causal interaction:
  All of the AMTIEs are zero if and only if all of the ATIEs are zero

- The AMTIEs can be estimated by first estimating the ATIEs
Higher-order Causal Interaction

- $J$ factorial treatments: $\mathbf{T} = (T_1, \ldots, T_J)$
- Assumptions:
  1. Full factorial design
     
     $$Y(t) \perp \perp \mathbf{T} \quad \text{and} \quad \Pr(\mathbf{T} = t) > 0 \quad \text{for all } t$$
  2. Independent treatment assignment
     
     $$T_j \perp \perp \mathbf{T}_{-j} \quad \text{for all } j$$
- Assumption 2 is not necessary for identification but considerably simplifies estimation
- We are interested in the $K$-way interaction where $K \leq J$
- We extend all the results for the 2-way interaction to this general case
Generalize the 2-way ATIE by marginalizing the other treatments $T^{1:2}$

$$\xi_{1:2}(t_1, t_2; t_{01}, t_{02}) \equiv \int \mathbb{E} \left\{ Y(t_1, t_2, T^{1:2}) - Y(t_{01}, t_2, T^{1:2}) ight. \\
\left. - Y(t_1, t_{02}, T^{1:2}) + Y(t_{01}, t_{02}, T^{1:2}) \right\} dF(T^{1:2})$$

In the literature, the 3-way ATIE is defined as

$$\xi_{1:3}(t_1, t_2, t_3; t_{01}, t_{02}, t_{03}) \equiv \xi_{1:2}(t_1, t_2; t_{01}, t_{02} \mid T_3 = t_3) - \xi_{1:2}(t_1, t_2; t_{01}, t_{02} \mid T_3 = t_{03})$$

Higher-order ATIEs are similarly defined sequentially

- This representation is based on the conditional effect interpretation
- Problem: the conditional effect of conditional effects!
Interactive Interpretation of the Higher-order ATIE

- We show that the higher-order ATIE also has an interactive effect interpretation.

- Example: 3-way ATIE, $\xi_{1:3}(t_1, t_2, t_3; t_{01}, t_{02}, t_{03})$, equals

\[
\tau_{1:3}(t_1, t_2, t_3; t_{01}, t_{02}, t_{03}) \quad \text{ATCE}
\]

\[
- \left\{ \xi_{1:2}(t_1, t_2; t_{01}, t_{02} \mid T_3 = t_{03}) + \xi_{2:3}(t_2, t_3; t_{02}, t_{03} \mid T_1 = t_{01}) \right. \\
+ \left. \xi_{1:3}(t_1, t_3; t_{01}, t_{03} \mid T_2 = t_{02}) \right\} \quad \text{sum of 2-way conditional ATIEs}
\]

\[
- \left\{ \tau_1(t_1, t_{02}, t_{03}; t_{01}, t_{02}, t_{03}) + \tau_2(t_{01}, t_2, t_{03}; t_{01}, t_{02}, t_{03}) \right. \\
+ \left. \tau_3(t_{01}, t_{02}, t_3; t_{01}, t_{02}, t_{03}) \right\} \quad \text{sum of (1-way) ATCEs}
\]

- Problems:
  1. Lower-order conditional ATIEs rather than lower-order ATIEs are used.
  2. $K$-way ATCE $\neq$ sum of all $K$-way and lower-order ATIEs.
  3. (We prove) Lack of invariance to the baseline conditions.
The $K$-way Average Marginal Treatment Interaction Effect

- **Definition:** the difference between the ATCE and the sum of lower-order AMTIEs
- **Interactive effect interpretation**
- **Example:** 3-way AMTIE, $\pi_{1:3}(t_1, t_2, t_3; t_{01}, t_{02}, t_{03})$, equals

\[
\begin{align*}
\tau_{1:3}(t_1, t_2, t_3; t_{01}, t_{02}, t_{03}) &= \text{ATCE} \\
- \left\{ \pi_{1:2}(t_1, t_2; t_{01}, t_{02}) + \pi_{2:3}(t_2, t_3; t_{02}, t_{03}) + \pi_{1:3}(t_1, t_3; t_{01}, t_{03}) \right\} &= \text{sum of 2-way AMTIEs} \\
- \left\{ \psi(t_1; t_{01}) + \psi(t_2; t_{02}) + \psi(t_3; t_{03}) \right\} &= \text{sum of (1-way) AMTEs}
\end{align*}
\]

- **Properties:**
  1. $K$-way ATCE = the sum of all $K$-way and lower-order AMTIEs
  2. Interval and order invariance to the baseline condition
  3. Derive the relationships between the AMTIEs and ATIEs for any order
Empirical Analysis of the Immigration Survey Experiment

- 5 factors \((\text{gender}^2, \text{education}^7, \text{origin}^{10}, \text{experience}^4, \text{plan}^4)\)
  1. full factorial design assumption
  2. computational tractability
- Matched-pair conjoint analysis: randomly choose one profile
- Binary outcome: whether a profile is selected
- Model with one-way, two-way, and three-way interaction terms
- \(p = 1,575\) and \(n = 6,980\)
- Curse of dimensionality \(\implies\) sparcity assumption
- Support vector machine with a lasso constraint (Imai & Ratkovic, 2013)
- Under-identified model that includes baseline conditions
- 99 non-zero and 1,476 zero coefficients
- Cross-validation for selecting a tuning parameter
- FindIt: Finding heterogeneous treatment effects
Three-way Effects:
- Education:Gender:Origin
- Education:Experience:Plan
- Education:Gender:Experience
- Education:Gender:Plan
- Gender:Experience:Plan
- Gender:Origin:Experience
- Gender:Origin:Plan

Two-way Effects:
- Origin:Experience
- Education:Origin
- Experience:Plan
- Education:Plan
- Education:Gender
- Gender:Plan
- Gender:Origin
- Gender:Experience

One-way Effects:
- Plan
- Education
- Origin
- Experience
- Gender

- **Range of AMTIEs:** importance of each factor and factor interaction
- **Sparcity-of-effects principle**
- gender appears to play a significant role in three-way interactions
- Exploration of level interactions
- \textbf{origin} \times \textbf{experience} interaction
- Baseline: India, None
- Only relative magnitude matters
- Little interaction for European origin
- Similar pattern for Mexico and Phillipines as well as Sudan and Somalia
Decomposing the Average Treatment Combination Effect

- **Two-way effect example (origin x experience):**

\[
\tau(\text{Somalia, 1-2 years; India, None})
\]

\[
-3.74
\]

\[
= \psi(\text{Somalia; India}) + \psi(1-2\text{years};\text{None}) + \pi(\text{Somalia, 1-2years; India, None})
\]

\[
-5.14 + 5.12 -3.72
\]

- **Three-way examples (education x gender x origin):**

\[
\tau(\text{Graduate, Male, India; Graduate, Female, India})
\]

\[
7.46
\]

\[
= \psi(\text{Male; Female}) + \pi(\text{Graduate, Male; Graduate, Female})
\]

\[
-0.77 -0.34
\]

\[
+ \pi(\text{Male, India; Female, India}) + \pi(\text{Graduate, Male, India; Graduate, Female, India})
\]

\[
1.56 + 7.01
\]
\[\tau(\text{High school, Male, Germany}; \text{High school, Female, Germany}) = -11.52\]
\[= \psi(\text{Male}; \text{Female}) + \pi(\text{High school, Male}; \text{High school, Female}) - 0.77 + \pi(\text{Male, Germany}; \text{Female, Germany}) - 3.34 + \pi(\text{High school, Male, Germany}; \text{High school, Female, Germany}) - 6.74\]
\[(n = 41; \ n = 56)\]
Concluding Remarks

- Interaction effects play an essential role in causal heterogeneity
  1. moderation
  2. causal interaction

- Two interpretations of causal interaction
  1. conditional effect interpretation (problematic in high dimension)
  2. interactive effect interpretation

- Average Marginal Treatment Interaction Effect
  1. interactive effect in high-dimension
  2. invariant to baseline condition
  3. enables effect decomposition
  4. $\Rightarrow$ effective analysis of interactions in high-dimension

- Estimation challenges in high dimension
  1. group lasso, hierarchical interaction
  2. post-selection inference
References


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