Statistical Analysis of Causal Mechanisms

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- This talk is based on the following paper:

  Imai, Kosuke, Luke Keele, and Teppei Yamamoto.
  "*Identification and Inference in Causal Mediation Analysis*"
  available at http://imai.princeton.edu/

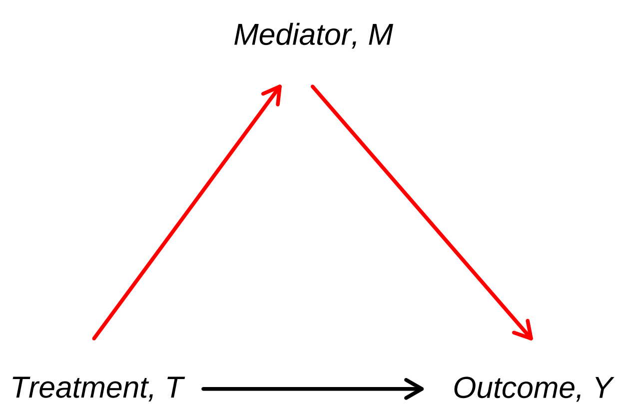
Randomized Experiments and Causal Mechanisms

- Causal inference is a central goal of social science and public policy research
- Randomized experiments are seen as **gold standard**
- Design and analyze observational studies to *replicate* experiments
- But, experiments are a **black box**
- Can only tell *whether* the treatment causally affects the outcome
- Not *how* and *why* the treatment affects the outcome
- Qualitative research uses process tracing

- How can quantitative research be used to identify causal mechanisms?

What This Talk is About

- **Goal:** Show how to identify causal mechanisms using statistics
- **Method:** Causal Mediation Analysis



- Direct and indirect effects; intermediate and intervening variables
- Popular among social psychologists (e.g., Baron and Kenny)

Common Practice in Social Science Research

- Regression

$$Y_i = \alpha + \beta T_i + \gamma M_i + \delta X_i + \epsilon_i$$

- Each coefficient is interpreted as a causal effect
- Sometimes, it's called marginal (or partial) effect
- Idea: increase T_i by one unit while holding M_i and X_i constant

- Post-treatment bias: if you change T_i, that may also change M_i
- Usual advice: only include causally prior (or pre-treatment) variables
- But, then you lose causal mechanisms!

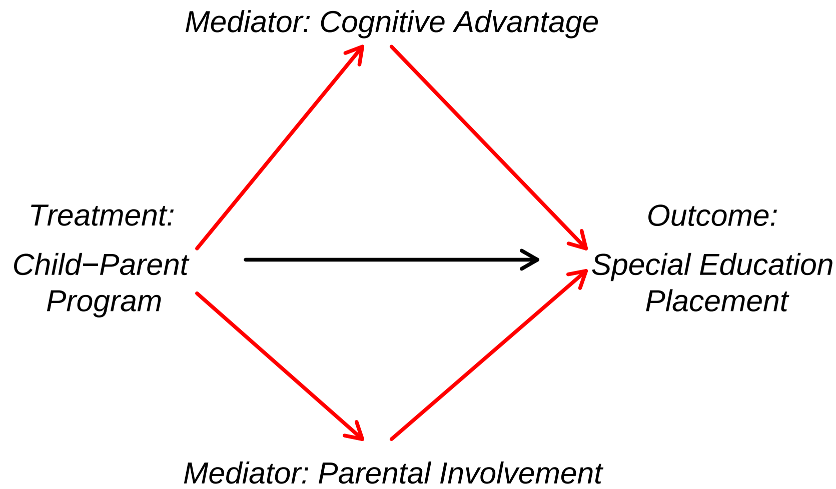
Example I: Early Childhood Intervention Programs

- Do early childhood intervention programs have long-term impact on educational outcomes? If so, how?
- Conyers et al. (2003) *Educational Evaluation and Policy Analysis*
- Observational data: The 1999 Chicago Longitudinal Study
- Low income predominantly African-American children in government-funded kindergarten programs

- Treatment: The Child-Parent Center preschool programs
  - structured half-day program for 3 and 4 years old
  - reading and writing activities
  - mandatory parental involvement
- Outcome: Participation in special education classes in grades 1–8

Hypotheses and Findings

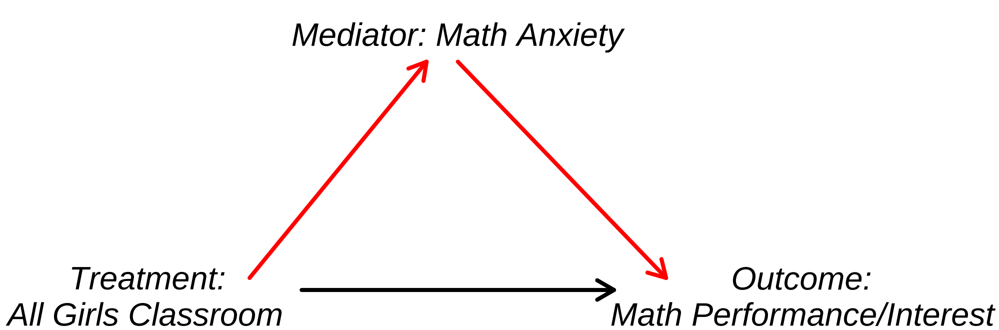
- Mediators: cognitive advantage vs. family support



- Findings:
  - The CPC program reduced the participation in special education
  - Cognitive advantage mediates the effect of the program, but family support does not

Example II: Girls-Only Curriculum in Math

- Classroom environment and math achievement
- Do girl-only classrooms affect math learning? If so, how?
- Shapka & Keating (2003) *American Educational Research Journal*
- Mediator: math anxiety



Design and Findings

- Two similar coeducational schools, one school offering girls-only math education
- Students filled out math attitudes surveys
- Outcomes: math achievement and sustained interest

- Large positive impact of girls-only curriculum
- Math anxiety does not seem to mediate this effect
- An alternative explanation: cooperative classroom environment?

Formal Statistical Framework of Causal Inference

- Units: $i = 1, \ldots, n$
- "Treatment": $T_i = 1$ if treated, $T_i = 0$ otherwise
- *Observed* outcome: Y_i
- Pre-treatment covariates: X_i
- Potential outcomes: $Y_i(1)$ and $Y_i(0)$ where $Y_i = Y_i(T_i)$

Student i	Teaching program T_i	Post-test score $Y_i(1)$	$Y_i(0)$	Gender X_{1i}	Pre-test score X_{2i}
1	1	88	?	M	77
2	0	?	76	M	72
3	0	?	85	F	82
⋮	⋮	⋮	⋮	⋮	⋮
n	1	89	?	M	78

- Causal effect: $Y_i(1) - Y_i(0)$

Identification of Causal Effects in Standard Settings

- Nonparametric identification: What can we learn from the data generating process alone?
- Average Treatment Effect (ATE):

$$\tau \equiv \mathbb{E}(Y_i(1) - Y_i(0))$$

- Ignorability (randomization, no omitted variable):

$$(Y_i(1), Y_i(0)) \perp\!\!\!\perp T_i \mid X_i$$

- Identification under ignorability:

$$\tau \;=\; \mathbb{E}(Y_i \mid T_i = 1, X_i) - \mathbb{E}(Y_i \mid T_i = 0, X_i)$$

- Relationship with the linear regression:

$$Y_i(T_i) = \alpha + \beta T_i + \gamma X_i + \epsilon_i$$

where ignorability implies $T_i \perp\!\!\!\perp \epsilon_i \mid X_i$

Notation for Causal Mediation Analysis

- Binary treatment: $T_i \in \{0, 1\}$
- Mediator: M_i
- Outcome: Y_i
- Observed covariates: X_i

- Potential mediators: $M_i(t)$ where $M_i = M_i(T_i)$
- Potential outcomes: $Y_i(t, m)$ where $Y_i = Y_i(T_i, M_i(T_i))$

Defining and Interpreting Causal Mediation Effects

- Total causal effect: $\tau_i \;\equiv\; Y_i(1, M_i(1)) - Y_i(0, M_i(0))$
- Causal mediation effects:

$$\delta_i(t) \;\equiv\; Y_i(t, M_i(1)) - Y_i(t, M_i(0))$$

- Change the mediator from $M_i(0)$ to $M_i(1)$ while holding the treatment constant at t
- Indirect effect of the treatment on the outcome through the mediator under treatment status t
- $Y_i(t, M_i(t))$ is observable but $Y_i(t, M_i(1-t))$ is not

- Direct effects:

$$\zeta_i(t) \;\equiv\; Y_i(1, M_i(t)) - Y_i(0, M_i(t))$$

- Change the treatment from 0 to 1 while holding the mediator constant at $M_i(t)$
- Total effect = mediation (indirect) effect + direct effect:

$$\tau_i \;=\; \delta_i(t) + \zeta_i(1-t)$$

- Quantities of interest: Average Causal Mediation Effects,

$$\bar{\delta}(t) \;\equiv\; \mathbb{E}(\delta_i(t)) \;=\; \mathbb{E}\{Y_i(t, M_i(1)) - Y_i(t, M_i(0))\}$$

The Main Identification Result

Assumption 1 (Sequential Ignorability)

$$\{Y_i(t,m), M_i(t)\} \perp\!\!\!\perp T_i \mid X_i,$$
$$Y_i(t,m) \perp\!\!\!\perp M_i \mid T_i, X_i$$

for $t = 0, 1$ and $m \in \mathcal{M}$.

Theorem 1 (Nonparametric Identification)

Under Assumption 1, for $t = 0, 1$,

$$\bar{\delta}(t) = (-1)^t \int \left\{ \int \mathbb{E}(Y_i \mid M_i, T_i = t, X_i)\, dP(M_i \mid T_i = 1-t, X_i) - \mathbb{E}(Y_i \mid T_i = t, X_i) \right\} dP(X_i)$$

Theoretical and Practical Implications

- Existing statistics literature concludes that an additional assumption is required for the identification of mediation effects
- However, sequential ignorability *alone* is sufficient
- Fit two nonparametric regressions:
    1. $\mu_{tm}(x) \equiv \mathbb{E}(Y_i \mid T_i = t, M_i = m, X_i = x)$
    2. $\lambda_{tm}(x) \equiv \Pr(M_i = m \mid T_i = t, X_i = x)$
- The plug-in estimator for a discrete mediator:

$$\hat{\delta}(t) = \frac{(-1)^t}{n} \left\{ \sum_{i=1}^{n} \sum_{m=0}^{J-1} \hat{\mu}_{tm}(X_i) \left(\hat{\lambda}_{1-t,m}(X_i) - \hat{\lambda}_{tm}(X_i) \right) \right\}.$$

Identification under Linear Structural Equation Model

Theorem 2 (Identification under LSEM)

Consider the following linear structural equation model

$$\begin{aligned} M_i &= \alpha_2 + \beta_2 T_i + \epsilon_{2i}, \\ Y_i &= \alpha_3 + \beta_3 T_i + \gamma M_i + \epsilon_{3i}. \end{aligned}$$

Under Assumption 1, the average causal mediation effects are identified as $\bar{\delta}(0) = \bar{\delta}(1) = \beta_2\gamma$.

- Run two (not three) regressions and multiply two coefficients!
- Direct effect: β_3
- Total effect: $\beta_2\gamma + \beta_3$
- Total effect could be zero even when mediation effects are not

Need for Sensitivity Analysis

- The sequential ignorability assumption is often too strong
- Need to assess the robustness of findings via sensitivity analysis
- Question: How large a departure from the key assumption must occur for the conclusions to no longer hold?
- Parametric and nonparametric sensitivity analysis by assuming

$$\{Y_i(t,m), M_i(t)\} \perp\!\!\!\perp T_i \mid X_i$$

but not

$$Y_i(t,m) \perp\!\!\!\perp M_i \mid T_i, X_i$$

Parametric Sensitivity Analysis

- Sensitivity parameter: $\rho \equiv \mathrm{Corr}(\epsilon_{2i}, \epsilon_{3i})$
- Existence of omitted variables leads to non-zero ρ
- Set ρ to different values and see how mediation effects change
- Can write estimated causal mediation effects as a function of ρ (and identifiable parameters)
- All you have to do: fit another regression

$$Y_i = \alpha_3^* + \beta_3^* T_i + \epsilon_{3i}^*$$

in addition to the previous two regressions:

$$\begin{aligned} M_i &= \alpha_2 + \beta_2 T_i + \epsilon_{2i} \\ Y_i &= \alpha_3 + \beta_3 T_i + \gamma M_i + \epsilon_{3i} \end{aligned}$$

- When do my results go away completely?
- $\bar{\delta}(t) = 0$ if and only if $\rho = \mathrm{Corr}(\epsilon_{2i}, \epsilon_{3i}^*)$ (easy to compute!)

An Alternative Interpretation of ρ

- A common omitted variable U_i

$$\begin{aligned} M_i &= \alpha_2 + \beta_2 T_i + \underbrace{\lambda_2 U_i + \epsilon'_{2i}}_{\epsilon_{2i}} \\ Y_i &= \alpha_3 + \beta_3 T_i + \gamma M_i + \underbrace{\lambda_3 U_i + \epsilon'_{3i}}_{\epsilon_{3i}} \end{aligned}$$

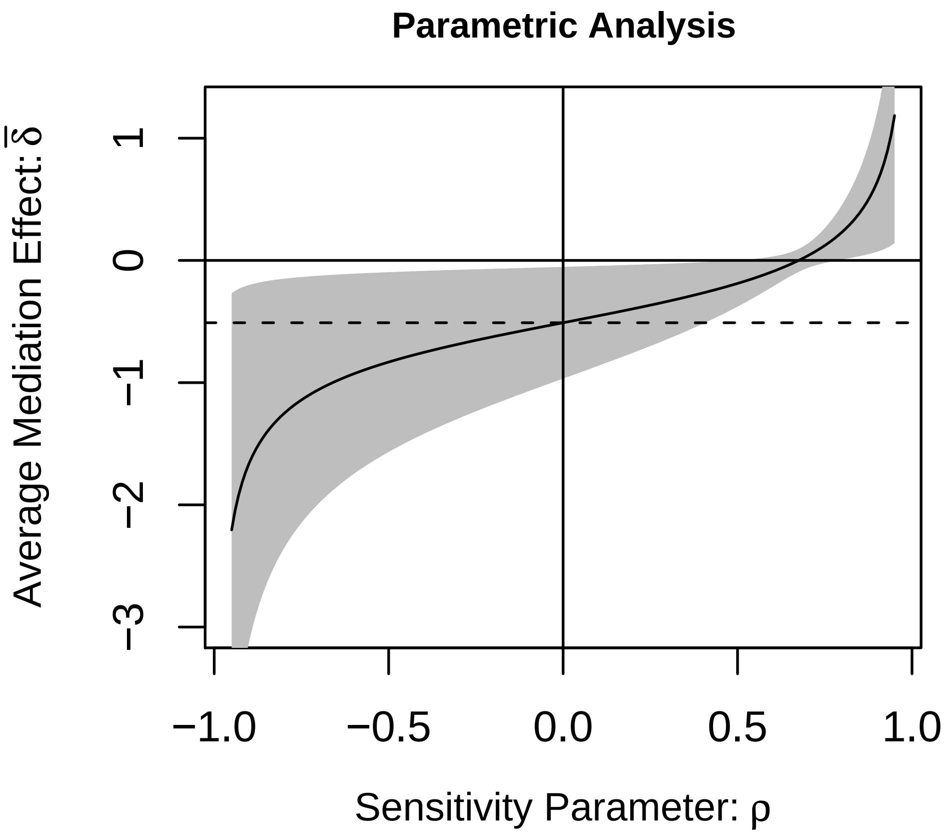
- How much does U_i matter?

$$R_M^2 = 1 - \frac{\mathrm{var}(\epsilon'_{2i})}{\mathrm{var}(\epsilon_{2i})} \quad \text{and} \quad R_Y^2 = 1 - \frac{\mathrm{var}(\epsilon'_{3i})}{\mathrm{var}(\epsilon_{3i})},$$

- The relationship:

$$\rho^2 = R_M^2 R_Y^2$$

Parametric Sensitivity Analysis



Concluding Remarks

- Quantitative analysis can be used to identify causal mechanisms!
- Estimate causal mediation effects rather than marginal effects
- Wide applications in social science disciplines

- Contributions to the statistics literature:
  1. Clarify assumptions
  2. Extend parametric method
  3. Develop nonparametric method
  4. Provide new sensitivity analysis