Identification and Sensitivity Analysis for Multiple Causal Mechanisms: Revisiting Evidence from Framing Experiments

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December 7, 2012
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Forthcoming in *Political Analysis*
Motivation

- Using causal mediation analysis to study causal mechanisms
- A fast-growing methodological focuses on a single mechanism:

\[ T \rightarrow M \rightarrow Y \]

- Identification, estimation, sensitivity analysis, new designs
- But, applied researchers analyze multiple mediators all the time
  - testing competing theories
  - adjusting for alternative mechanisms (post-treatment confounders)
- What does it take to analyze multiple mediators?
Quantity of interest = The average indirect effect with respect to $M$

$W$ represents the alternative observed mediators.

Left: Assumes independence between the two mechanisms

Right: Allows $M$ to be affected by the other mediators $W$

$W$ also represent post-treatment confounders between $M$ and $Y$

Applied work often assumes the independence of mechanisms
Our Contributions

- Analyze multiple mediators under the sequential ignorability assumption that allow for post-treatment confounders
- Use a flexible and yet interpretable model: semi-parametric random coefficient linear structural equation model
- Identification under the homogeneous interaction assumption
- Sensitivity analysis for possible heterogeneity in the degree of treatment-mediator interaction
- Extension to new experimental designs to avoid the sequential ignorability assumption
Outline of the Talk

1. Introduction
2. Framing Experiments in Political Psychology
3. Identification of Independent Multiple Mechanisms
4. Identification of Causally Related Multiple Mechanisms
5. Empirical Applications
6. Extensions, Software, and Conclusion
Issue framing may affect how individuals perceive the issue and change attitudes and behavior (Tversky and Kahneman 1981)

Political psychology: How does framing of political issues affect public opinions?

**Example 1:** Druckman and Nelson (2003) \((N = 261)\)

- Treatment: News paper article on a proposed election campaign finance reform, emphasizing either its positive or negative aspect
- Outcome: Support for the proposed reform
- Primary mediator: Perceived importance of free speech
- Alternative (confounding) mediator: Belief about the impact of the proposed reform
- Original analysis finds the importance mechanism to be significant, implicitly assuming its independence from beliefs
Original Analysis Assumes Independent Mechanisms

Druckman and Nelson, p.738
Example 2: Slothuus (2008) \((N = 408)\)
- Essentially the same study as Druckman and Nelson (2003)
- Treatment: News paper article on a social welfare reform bill
- Outcome: Opinion about the bill
- Primary mediator: Issue importance
- Alternative mediator: Belief content

Example 3: Brader, Valentino and Suhay (2008) \((N = 354)\)
- Treatment: News article about immigration, stressing either positive or negative aspects and featuring different ethnicities
- Outcome: Attitude toward increased immigration
- Primary mediator: Anxiety
- Alternative mediator: Perceived harm of increased immigration
Causal Mediation Analysis with a Single Mediator

- We first review the results for a single mediator (Imai et al. 2011)

- Causal mediation effect (indirect effect):
  \[
  \delta_i(t) \equiv Y_i(t, M_i(1)) - Y_i(t, M_i(0))
  \]

- Natural direct effect:
  \[
  \zeta_i(t) \equiv Y_i(1, M_i(t)) - Y_i(0, M_i(t))
  \]

- Total causal effect:
  \[
  \tau_i \equiv Y_i(1, M_i(1)) - Y_i(0, M_i(0)) = \delta_i(t) + \zeta_i(1 - t)
  \]

- The average indirect effect (\(\bar{\delta}(t) \equiv \mathbb{E}(\delta_i(t))\)) is nonparametrically identified under the (strong) sequential ignorability assumption:
  \[
  \{Y_i(t, m), M_i(t')\} \perp \perp T_i \mid X_i = x \quad (1)
  \]
  \[
  Y_i(t', m) \perp \perp M_i \mid T_i = t, X_i = x \quad (2)
  \]

for any value of \(x, t, t', m\) and every unit \(i\).
Causally Independent Alternative Mediators

- The existence of post-treatment confounders is precluded
- Equivalent to assuming that other mediators are independent of the primary mediator
- Formally, make those alternative mediators $W$ explicit:

  Potential mediators: $M_i(t)$ and $W_i(t)$
  Potential outcomes: $Y_i(t, m, w)$

  Note that $M_i(t)$ is only defined with respect to $t$ not $w$

- The indirect and natural direct effects:

  $\delta_i^M(t) \equiv Y_i(t, M_i(1), W_i(t)) - Y_i(t, M_i(0), W_i(t))$
  $\delta_i^W(t) \equiv Y_i(t, M_i(t), W_i(1)) - Y_i(t, M_i(t), W_i(0))$
  $\zeta_i(t, t') \equiv Y_i(1, M_i(t), W_i(t')) - Y_i(0, M_i(t), W_i(t'))$

- These sum up to the total effect, as expected:

  $\tau_i = \delta_i^M(t) + \delta_i^W(1 - t) + \zeta_i(1 - t, t)$
Identification of Independent Multiple Mechanisms

The average indirect effects \( \bar{\delta}^M(t) \equiv \mathbb{E}(\delta_i^M(t)) \) and \( \bar{\delta}^W(t) \equiv \mathbb{E}(\delta_i^W(t)) \) are nonparametrically identified under the following assumption:

Assumption 1

\[
\begin{align*}
\{ Y_i(t, m, w), & M_i(t'), W_i(t'') \} \perp T_i \mid X_i = x, \\
Y_i(t', m, W_i(t')) & \perp M_i \mid T_i = t, X_i = x, \\
Y_i(t', M_i(t'), w) & \perp W_i \mid T_i = t, X_i = x,
\end{align*}
\] (3)

for any \( x, t, t', m, w \).

Note that this is essentially the same assumption as Imai et al.'s sequential ignorability — only difference is \( W_i(t) \) is explicitly written out.
Unpacking the Standard Path-Analytic Approach

- Applied social scientists often use the following model:
  \[ M_i = \alpha_M + \beta_M T_i + \xi_M^T X_i + \epsilon_iM \]
  \[ W_i = \alpha_W + \beta_W T_i + \xi_W^T X_i + \epsilon_iW \]
  \[ Y_i = \alpha_3 + \beta_3 T_i + \gamma M_i + \theta^T W_i + \xi_3^T X_i + \epsilon_i3 \]

- The mediation effects are then estimated as \( \hat{\beta}_M \hat{\gamma} \) for \( M \) and \( \hat{\beta}_W \hat{\theta} \) for \( W \).

- We can show that these are consistent for \( \bar{\delta}_i^M \) and \( \bar{\delta}_i^W \) under the above assumption and linearity.

- However, because of the assumed independence between mechanisms, analyzing one mechanism at a time will also be valid, e.g.,
  \[ M_i = \alpha_2 + \beta_2 T_i + \xi_2^T X_i + \epsilon_i2 \]
  \[ Y_i = \alpha_3 + \beta_3 T_i + \gamma M_i + \xi_3^T X_i + \epsilon_i3 \]
Now we allow $W$ to influence both $M$ and $Y$:

Potential mediators: $W_i(t)$ and $M_i(t, w)$
Potential outcomes: $Y_i(t, m, w)$

The indirect and natural direct effects w.r.t. primary mediator:

$$
\delta_i(t) \equiv Y_i(t, M_i(1, W_i(1)), W_i(t)) - Y_i(t, M_i(0, W_i(0)), W_i(t))
$$

$$
\zeta_i(t) \equiv Y_i(1, M_i(t, W_i(t)), W_i(1)) - Y_i(0, M_i(t, W_i(t)), W_i(0))
$$

These again sum up to the total effect:

$$
\tau_i \equiv Y_i(1, M_i(1, W_i(1)), W_i(1)) - Y_i(0, M_i(0, W_i(0)), W_i(0))
= \delta_i(t) + \zeta_i(1 - t)
$$
Consider the (weak) sequential ignorability assumption, a special case of Robins’ FRCISTG:

Assumption 2

\[
\begin{align*}
\{ Y_i(t, m, w), M_i(t, w), W_i(t) \} & \perp T_i \mid X_i = x \\
\{ Y_i(t, m, w), M_i(t, w) \} & \perp W_i \mid T_i = t, X_i = x \\
\{ Y_i(t, m, w) \} & \perp M_i \mid W_i(t) = w, T_i = t, X_i = x
\end{align*}
\]

for any \( t, m, w, x \).

- Unconfoundedness of \( M_i \) conditional on both pre-treatment (\( X_i \)) and observed post-treatment (\( W_i \)) confounders

- Corresponds to sequential randomization unlike Assumption 1

- Robins (2003) shows that we need the no \( T \times M \) interaction assumption for the nonparametric identification of \( \bar{\delta}(t) \) under Assumption 2:

\[
Y_i(1, m, W_i(1)) - Y_i(0, m, W_i(0)) = Y_i(1, m', W_i(1)) - Y_i(0, m', W_i(0))
\]
The Proposed Framework

- Problem: The no interaction assumption is too strong in most applications
  (e.g. Does the effect of perceived issue importance invariant across frames?)

- We use a varying-coefficient linear structural equations model to:
  1. Allow for homogeneous interaction for point identification
  2. Develop a sensitivity analysis in terms of the degree of heterogeneity in the interaction effect

- Consider the following model:
  \[ M_i(t, w) = \alpha_2 + \beta_{2i} t + \xi_{2i}^T w + \mu_{2i}^T tw + \lambda_{2i}^T x + \epsilon_{2i}, \]
  \[ Y_i(t, m, w) = \alpha_3 + \beta_{3i} t + \gamma_i m + \kappa_i tm + \xi_{3i}^T w + \mu_{3i}^T tw + \lambda_{3i}^T x + \epsilon_{3i}, \]

  where \( \mathbb{E}(\epsilon_{2i}) = \mathbb{E}(\epsilon_{3i}) = 0 \)

- Allows for dependence of \( M \) on \( W \)

- Coefficients are allowed to vary arbitrarily across units
Sensitivity Analysis w.r.t. Interaction Heterogeneity

- Note that the model can be rewritten as:
  \[ M_i(t, w) = \alpha_2 + \beta_2 t + \xi_2^\top w + \mu_2^\top tw + \lambda_2^\top x + \eta_2i(t, w), \]
  \[ Y_i(t, m, w) = \alpha_3 + \beta_3 t + \gamma m + \kappa tm + \xi_3^\top w + \mu_3^\top tw + \lambda_3^\top x + \eta_3i(t, m, w), \]
  where \( \beta_2 = \mathbb{E}(\beta_{2i}), \) etc.

- Assumption 2 implies
  \[ \mathbb{E}(\eta_{2i}(T_i, W_i) \mid X_i, T_i, W_i) = \mathbb{E}(\eta_{3i}(T_i, M_i, W_i) \mid X_i, T_i, W_i, M_i) = 0 \]
  The mean coefficients \( \beta_2, \) etc. can thus be estimated without bias.

- We can show that \( \bar{\delta}(t) \) and \( \bar{\zeta}(t) \) can be written as
  \[ \bar{\delta}(t) = \bar{\tau} - \bar{\zeta}(1 - t) \]
  \[ \bar{\zeta}(t) = \beta_3 + \kappa \mathbb{E}(M_i \mid T_i = t) + \rho_t \sigma \sqrt{\mathbb{V}(M_i \mid T_i = t)} \]
  \[ + (\xi_3 + \mu_3)^\top \mathbb{E}(W_i \mid T_i = 1) - \xi_3^\top \mathbb{E}(W_i \mid T_i = 0) \]
  where \( \rho_t = \text{Corr}(M_i(t, W_i(t)), \kappa_i) \) and \( \sigma = \sqrt{\mathbb{V}(\kappa_i)} \) are the only unidentified quantities.
Remarks on the Proposed Sensitivity Analysis

- The two sensitivity parameters:
  - \( \rho_t \): Roughly, direction of the interaction (hard to interpret)
  - \( \sigma \): Degree of heterogeneity in the treatment-mediator interaction

- We therefore set \( \rho_t \in [-1, 1] \) and examine the sharp bounds on \( \bar{\delta}(t) \) as functions of \( \sigma \)

- Consider the following homogeneous interaction assumption:
  \[
  Y_i(1, m, W_i(1)) - Y_i(0, m, W_i(0)) = B_i + Cm
  \]
  This implies \( \sigma = 0 \) and therefore \( \bar{\delta}(t) \) and \( \bar{\zeta}(t) \) are identified

- An alternative formulation using the coefficients of determination:
  \[
  R^2_* = \frac{\text{V}(\tilde{\kappa}_i T_i M_i)}{\text{V}(\eta_{3i}(T_i, M_i, W_i))} \quad \text{and} \quad \tilde{R}^2 = \frac{\text{V}(\tilde{\kappa}_i T_i M_i)}{\text{V}(Y_i)}
  \]
  - One-to-one relationship with \( \sigma \):
    \[
    \sigma = \sqrt{\text{V}(\eta_{3i}(T_i, M_i, W_i))} R^2_* / \text{E}(T_i M_i^2) = \sqrt{\text{V}(Y_i) \tilde{R}^2 / \text{E}(T_i M_i^2)}
    \]
  - Implies an upper bound on \( \sigma \): \( 0 < \sigma < \sqrt{\text{V}(\eta_{3i}(T_i, M_i, W_i)) / \text{E}(T_i M_i^2)} \)
Weakly significant average indirect effects ([0.025, 0.625]), accounting for 28.6 percent of the total effect.

Moderate degree of sensitivity to the mediator exogeneity ($\bar{\delta} = 0$ when $\rho = -0.43$ or $\tilde{R}_M^2 \tilde{R}_Y^2 = 0.078$).

Concern (both theoretical and empirical) that the importance mechanism may be affected by the belief content mechanism.
The point estimate is similar with slightly wider CI ($[-0.021, 0.648]$).

Lower bound on $\bar{\delta}$ equals zero when $\sigma = 0.195$, or 51% of its upper bound.

This translates to the interaction heterogeneity explaining 15.9% of the variance of the outcome variable.
Analysis under the Independence Assumption

### Slothuus (2008)

- **Point Estimates**
  - Average ($\bar{\delta}$)
  - Treated ($\bar{\delta}_1$)
  - Control ($\bar{\delta}_0$)
  - Total ($\bar{\tau}$)

- **Sensitivity with Respect to Error Correlation**
  - $\bar{\delta}(\rho)$
  - $R^2_{\text{M}}$
  - $R^2_{\text{Y}}$

### Brader, Valentino & Suhay (2008)

- **Point Estimates**
  - Average ($\bar{\delta}$)
  - Treated ($\bar{\delta}_1$)
  - Control ($\bar{\delta}_0$)
  - Total ($\bar{\tau}$)

- **Sensitivity with Respect to Error Correlation**
  - $\bar{\delta}(\rho)$
  - $R^2_{\text{M}}$
  - $R^2_{\text{Y}}$
Analysis without the Independence Assumption

Slothuus (2008)

Point Estimates

Average (\(\bar{\delta}\))
Treated (\(\bar{\delta}_1\))
Control (\(\bar{\delta}_0\))
Total (\(\bar{\tau}\))

Average Causal Mediation Effects

Sensitivity with Respect to Interaction Heterogeneity

\(\bar{\delta}(\sigma)\)

Sensitivity with Respect to Importance of Interaction

\(\bar{\delta}(\tilde{R}^2)\)

Brader, Valentino and Suhay (2008)

Point Estimates

Average (\(\bar{\delta}\))
Treated (\(\bar{\delta}_1\))
Control (\(\bar{\delta}_0\))
Total (\(\bar{\tau}\))

Average Causal Mediation Effects

Sensitivity with Respect to Interaction Heterogeneity

\(\bar{\delta}(\sigma)\)

Sensitivity with Respect to Importance of Interaction

\(\bar{\delta}(\tilde{R}^2)\)
Extensions to New Experimental Designs

- The above analysis assumes (weak) sequential ignorability
- All pre- and post-treatment confounders are assumed to be observed
- Possible existence of unobserved confounders

- Randomized experiment to manipulate the primary mediator
- Natural experiments where the primary mediator is as-if random

- Parallel design:
  - 1. Randomize treatment
  - 2. Randomize both treatment and mediator

- Parallel encouragement design:
  - imperfect manipulation of the mediator
  - a randomized instrument for the mediator
Semi-parametric random coefficient linear model:

\[ M_i(t) = \alpha_2 + \beta_{2i}t + \epsilon_{2i} \]
\[ Y_i(t, m) = \alpha_3 + \beta_{3i}t + \gamma_im + \kappa_itm + \epsilon_{3i}, \]

Quantities of interest:

\[ \bar{\delta}(t) = \beta_1 - \bar{\zeta}(1 - t) \]
\[ \bar{\zeta}(t) = \beta_3 + (\alpha_2 + \beta_{2t})\kappa + \rho_t\sigma\sqrt{\mathbb{V}(M_i \mid T_i = t, D_i = 0)} \]

Sensitivity analysis via \( \rho_t \) and \( \sigma \)
Mediator model changes to

\[ M_i(t, z) = \alpha_2 + \beta_{2i}t + \lambda_i z + \theta_i tz + \epsilon_{2i} \]

where \( z \) represents the value of randomized encouragement.

Outcome model stays identical to that for parallel design

\[ Y_i(t, m) = \alpha_3 + \beta_{3i}t + \gamma_i m + \kappa_i tm + \epsilon_{3i}, \]

Two-stage least squares model

Sensitivity analysis via \( \rho_{tz} \) and \( \sigma \)
Implementation via R Package `mediation`

An example syntax:

```r
## pre-treatment covariates
Xnames <- c("age", "educ", "gender", "income")
## fit the model
m.med <- multimed(outcome = "immigr", med.main = "emo",
                   med.alt = "p_harm", treat = "treat",
                   covariates = Xnames,
                   data = framing, sims = 1000)

## summary
summary(m.med)
## point estimate under homogenous interaction
plot(m.med, type = "point")
## sensitivity analysis based on R2
plot(m.med, type = "R2-total")
```

For the parallel design, set `design = "parallel"` in `multimed()`
Concluding Remarks and Future Research

- Causal mediation analysis with multiple mediators is complicated!
- Critical issue: relationships among mediators
  1. causal ordering
  2. causal dependence
- (Sequential) ignorability is not sufficient:
  - Randomization of mediator does not solve the problem
  - Importance of heterogeneous treatment
  - Treatment-mediator interaction
- What explains heterogeneous interaction effects?
- Can we adjust for those factors when designing and analyzing your study?
- Much methodological work remains to be done:
  - causal mediation in multi-level settings
  - causal mediation in longitudinal settings