Identification and Sensitivity Analysis for Multiple Causal Mechanisms: Revisiting Evidence from Framing Experiments

Kosuke Imai

Princeton University

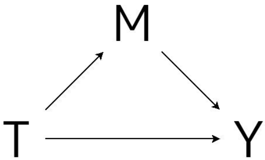
December 7, 2012
University of Wisconsin, Madison

Joint work with Teppei Yamamoto (MIT)

Forthcoming in *Political Analysis*

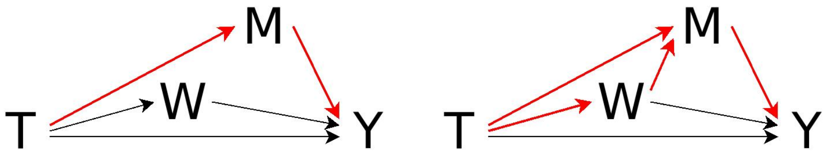
Motivation

- Using causal mediation analysis to study causal mechanisms
- A fast-growing methodological focuses on a single mechanism:



- Identification, estimation, sensitivity analysis, new designs
- But, applied researchers analyze multiple mediators all the time
  - testing competing theories
  - adjusting for alternative mechanisms (post-treatment confounders)
- What does it take to analyze multiple mediators?

Causally Independent vs. Dependent Mechanisms



- Quantity of interest $=$ The average indirect effect with respect to M
- W represents the alternative observed mediators

- Left: Assumes independence between the two mechanisms
- Right: Allows M to be affected by the other mediators W
- W also represent post-treatment confounders between M and Y

- Applied work often assumes the independence of mechanisms

Our Contributions

- Analyze multiple mediators under the sequential ignorability assumption that allow for post-treatment confounders
- Use a flexible and yet interpretable model: semi-parametric random coefficient linear structural equation model
- Identification under the homogeneous interaction assumption
- Sensitivity analysis for possible heterogeneity in the degree of treatment-mediator interaction
- Extension to new experimental designs to avoid the sequential ignorability assumption

Outline of the Talk

1. Introduction
2. Framing Experiments in Political Psychology
3. Identification of Independent Multiple Mechanisms
4. Identification of Causally Related Multiple Mechanisms
5. Empirical Applications
6. Extensions, Software, and Conclusion

Running Examples: Framing Experiments I

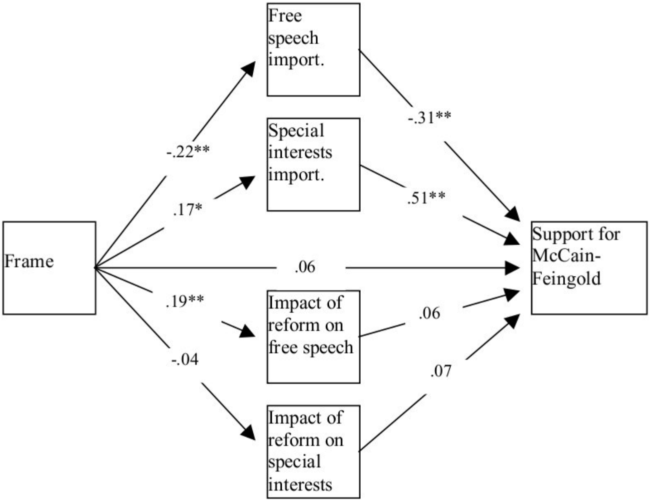
- Issue framing may affect how individuals perceive the issue and change attitudes and behavior (Tversky and Kahneman 1981)
- Political psychology: How does framing of political issues affect public opinions?

Example 1: Druckman and Nelson (2003) ($N = 261$)

- Treatment: News paper article on a proposed election campaign finance reform, emphasizing either its positive or negative aspect
- Outcome: Support for the proposed reform
- Primary mediator: Perceived importance of free speech
- Alternative (confounding) mediator: Belief about the impact of the proposed reform
- Original analysis finds the importance mechanism to be significant, implicitly assuming its independence from beliefs

Original Analysis Assumes Independent Mechanisms

Druckman and Nelson, p.738



Running Examples: Framing Experiments II and III

Example 2: Slothuus (2008) ($N = 408$)

- Essentially the same study as Druckman and Nelson (2003)
- Treatment: News paper article on a social welfare reform bill
- Outcome: Opinion about the bill
- Primary mediator: Issue importance
- Alternative mediator: Belief content

Example 3: Brader, Valentino and Suhay (2008) ($N = 354$)

- Treatment: News article about immigration, stressing either positive or negative aspects and featuring different ethnicities
- Outcome: Attitude toward increased immigration
- Primary mediator: Anxiety
- Alternative mediator: Perceived harm of increased immigration

Causal Mediation Analysis with a Single Mediator

- We first review the results for a single mediator (Imai et al. 2011)
- Causal mediation effect (indirect effect):

$$\delta_i(t) \equiv Y_i(t, M_i(1)) - Y_i(t, M_i(0))$$

- Natural direct effect:

$$\zeta_i(t) \equiv Y_i(1, M_i(t)) - Y_i(0, M_i(t))$$

- Total causal effect:

$$\tau_i \equiv Y_i(1, M_i(1)) - Y_i(0, M_i(0)) = \delta_i(t) + \zeta_i(1-t)$$

- The average indirect effect ($\bar{\delta}(t) \equiv \mathbb{E}(\delta_i(t))$) is nonparametrically identified under the (strong) sequential ignorability assumption:

$$\{Y_i(t, m), M_i(t')\} \perp\!\!\!\perp T_i \mid X_i = x \quad (1)$$

$$Y_i(t', m) \perp\!\!\!\perp M_i \mid T_i = t, X_i = x \quad (2)$$

for any value of x, t, t', m and every unit i.

Causally Independent Alternative Mediators

- The existence of post-treatment confounders is precluded
- Equivalent to assuming that other mediators are independent of the primary mediator

- Formally, make those alternative mediators W explicit:

$$\text{Potential mediators:} \quad M_i(t) \text{ and } W_i(t)$$
$$\text{Potential outcomes:} \quad Y_i(t, m, w)$$

  Note that $M_i(t)$ is only defined with respect to t not w

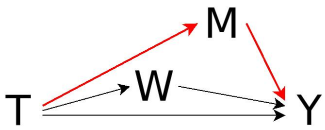
- The indirect and natural direct effects:

$$\begin{aligned}
\delta_i^M(t) &\equiv Y_i(t, M_i(1), W_i(t)) - Y_i(t, M_i(0), W_i(t)) \\
\delta_i^W(t) &\equiv Y_i(t, M_i(t), W_i(1)) - Y_i(t, M_i(t), W_i(0)) \\
\zeta_i(t, t') &\equiv Y_i(1, M_i(t), W_i(t')) - Y_i(0, M_i(t), W_i(t'))
\end{aligned}$$

- These sum up to the total effect, as expected:

$$\tau_i = \delta_i^M(t) + \delta_i^W(1-t) + \zeta_i(1-t, t)$$

Identification of Independent Multiple Mechanisms



- The average indirect effects ($\bar{\delta}^M(t) \equiv \mathbb{E}(\delta_i^M(t))$ and $\bar{\delta}^W(t) \equiv \mathbb{E}(\delta_i^W(t))$) are nonparametrically identified under the following assumption:

Assumption 1

$$\{Y_i(t, m, w), M_i(t'), W_i(t'')\} \perp\!\!\!\perp T_i \mid X_i = x, \quad (3)$$

$$Y_i(t', m, W_i(t')) \perp\!\!\!\perp M_i \mid T_i = t, X_i = x, \quad (4)$$

$$Y_i(t', M_i(t'), w) \perp\!\!\!\perp W_i \mid T_i = t, X_i = x, \quad (5)$$

for any x, t, t', m, w.

- Note that this is essentially the same assumption as Imai et al.'s sequential ignorability — only difference is $W_i(t)$ is explicitly written out

Unpacking the Standard Path-Analytic Approach

- Applied social scientists often use the following model:

$$\begin{aligned} M_i &= \alpha_M + \beta_M T_i + \xi_M^\top X_i + \epsilon_{iM} \\ W_i &= \alpha_W + \beta_W T_i + \xi_W^\top X_i + \epsilon_{iW} \\ Y_i &= \alpha_3 + \beta_3 T_i + \gamma M_i + \theta^\top W_i + \xi_3^\top X_i + \epsilon_{i3} \end{aligned}$$

- The mediation effects are then estimated as $\hat{\beta}_M \hat{\gamma}$ for M and $\hat{\beta}_W \hat{\theta}$ for W
- We can show that these are consistent for $\bar{\delta}_i^M$ and $\bar{\delta}_i^W$ under the above assumption and linearity
- However, because of the assumed independence between mechanisms, analyzing one mechanism at a time will also be valid, e.g.,

$$\begin{aligned} M_i &= \alpha_2 + \beta_2 T_i + \xi_2^\top X_i + \epsilon_{i2} \\ Y_i &= \alpha_3 + \beta_3 T_i + \gamma M_i + \xi_3^\top X_i + \epsilon_{i3} \end{aligned}$$

Causally Related Multiple Mechanisms

- Now we allow W to influence both M and Y:

$$\text{Potential mediators:} \quad W_i(t) \text{ and } M_i(t, \color{red}{w}\color{black})$$
$$\text{Potential outcomes:} \quad Y_i(t, m, w)$$

- The indirect and natural direct effects w.r.t. primary mediator:

$$\begin{aligned} \delta_i(t) &\equiv Y_i(t, M_i(1, W_i(1)), W_i(t)) - Y_i(t, M_i(0, W_i(0)), W_i(t)) \\ \zeta_i(t) &\equiv Y_i(1, M_i(t, W_i(t)), W_i(1)) - Y_i(0, M_i(t, W_i(t)), W_i(0)) \end{aligned}$$

- These again sum up to the total effect:

$$\begin{aligned} \tau_i &\equiv Y_i(1, M_i(1, W_i(1)), W_i(1)) - Y_i(0, M_i(0, W_i(0)), W_i(0)) \\ &= \delta_i(t) + \zeta_i(1 - t) \end{aligned}$$

Identification of Causally Related Mechanisms

- Consider the (weak) sequential ignorability assumption, a special case of Robins' FRCISTG:

Assumption 2

$$\begin{aligned} \{Y_i(t,m,w), M_i(t,w), W_i(t)\} &\perp\!\!\!\perp T_i \mid X_i = x \\ \{Y_i(t,m,w), M_i(t,w)\} &\perp\!\!\!\perp W_i \mid T_i = t,\ X_i = x \\ \{Y_i(t,m,w)\} &\perp\!\!\!\perp M_i \mid W_i(t) = w,\ T_i = t,\ X_i = x \end{aligned}$$

for any t, m, w, x.

- Unconfundedness of M_i conditional on both pre-treatment (X_i) and observed post-treatment (W_i) confounders
- Corresponds to sequential randomization unlike Assumption 1
- Robins (2003) shows that we need the no $T \times M$ interaction assumption for the nonparametric identification of $\bar{\delta}(t)$ under Assumption 2:

$$Y_i(1, m, W_i(1)) - Y_i(0, m, W_i(0)) \;=\; Y_i(1, m', W_i(1)) - Y_i(0, m', W_i(0))$$

The Proposed Framework

- Problem: The no interaction assumption is too strong in most applications
  (e.g. Does the effect of perceived issue importance invariant across frames?)
- We use a varying-coefficient linear structural equations model to:
  1. Allow for homogeneous interaction for point identification
  2. Develop a sensitivity analysis in terms of the degree of heterogeneity in the interaction effect

- Consider the following model:

$$\begin{aligned} M_i(t, w) &= \alpha_2 + \beta_{2i} t + \xi_{2i}^\top w + \mu_{2i}^\top t w + \lambda_{2i}^\top x + \epsilon_{2i}, \\ Y_i(t, m, w) &= \alpha_3 + \beta_{3i} t + \gamma_i m + \kappa_i t m + \xi_{3i}^\top w + \mu_{3i}^\top t w + \lambda_{3i}^\top x + \epsilon_{3i}, \end{aligned}$$

  where $\mathbb{E}(\epsilon_{2i}) = \mathbb{E}(\epsilon_{3i}) = 0$
- Allows for dependence of M on W
- Coefficients are allowed to vary arbitrarily across units

Sensitivity Analysis w.r.t. Interaction Heterogeneity

- Note that the model can be rewritten as:

$$\begin{aligned} M_i(t, w) &= \alpha_2 + \beta_2 t + \xi_2^\top w + \mu_2^\top tw + \lambda_2^\top x + \eta_{2i}(t, w), \\ Y_i(t, m, w) &= \alpha_3 + \beta_3 t + \gamma m + \kappa tm + \xi_3^\top w + \mu_3^\top tw + \lambda_3^\top x + \eta_{3i}(t, m, w), \end{aligned}$$

  where $\beta_2 = \mathbb{E}(\beta_{2i})$, etc.

- Assumption 2 implies

$$\mathbb{E}(\eta_{2i}(T_i, W_i) \mid X_i, T_i, W_i) = \mathbb{E}(\eta_{3i}(T_i, M_i, W_i) \mid X_i, T_i, W_i, M_i) = 0$$

  The mean coefficients β_2, etc. can thus be estimated without bias

- We can show that $\bar{\delta}(t)$ and $\bar{\zeta}(t)$ can be written as

$$\begin{aligned} \bar{\delta}(t) &= \bar{\tau} - \bar{\zeta}(1 - t) \\ \bar{\zeta}(t) &= \beta_3 + \kappa \mathbb{E}(M_i \mid T_i = t) + \rho_t \sigma \sqrt{\mathbb{V}(M_i \mid T_i = t)} \\ &\quad + (\xi_3 + \mu_3)^\top \mathbb{E}(W_i \mid T_i = 1) - \xi_3^\top \mathbb{E}(W_i \mid T_i = 0) \end{aligned}$$

  where $\rho_t = \mathrm{Corr}(M_i(t, W_i(t)), \kappa_i)$ and $\sigma = \sqrt{\mathbb{V}(\kappa_i)}$ are the only unidentified quantities

Remarks on the Proposed Sensitivity Analysis

- The two sensitivity parameters:
  - ρ_t: Roughly, direction of the interaction (hard to interpret)
  - σ: Degree of heterogeneity in the treatment-mediator interaction
- We therefore set $\rho_t \in [-1, 1]$ and examine the sharp bounds on $\bar{\delta}(t)$ as functions of σ
- Consider the following homogeneous interaction assumption:

$$Y_i(1, m, W_i(1)) - Y_i(0, m, W_i(0)) \;=\; B_i + Cm$$

  This implies $\sigma = 0$ and therefore $\bar{\delta}(t)$ and $\bar{\zeta}(t)$ are identified
- An alternative formulation using the coefficients of determination:

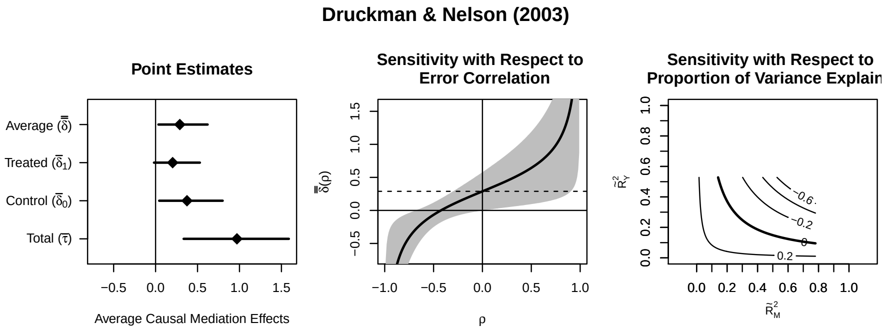
$$R^{2*} \;=\; \frac{\mathbb{V}(\tilde{\kappa}_i T_i M_i)}{\mathbb{V}(\eta_{3i}(T_i, M_i, W_i))} \quad \text{and} \quad \widetilde{R}^2 \;=\; \frac{\mathbb{V}(\tilde{\kappa}_i T_i M_i)}{\mathbb{V}(Y_i)}$$

  - One-to-one relationship with σ:

$$\sigma \;=\; \sqrt{\mathbb{V}(\eta_{3i}(T_i, M_i, W_i))R^{2*}/\mathbb{E}(T_i M_i^2)} \;=\; \sqrt{\mathbb{V}(Y_i)\widetilde{R}^2/\mathbb{E}(T_i M_i^2)}$$

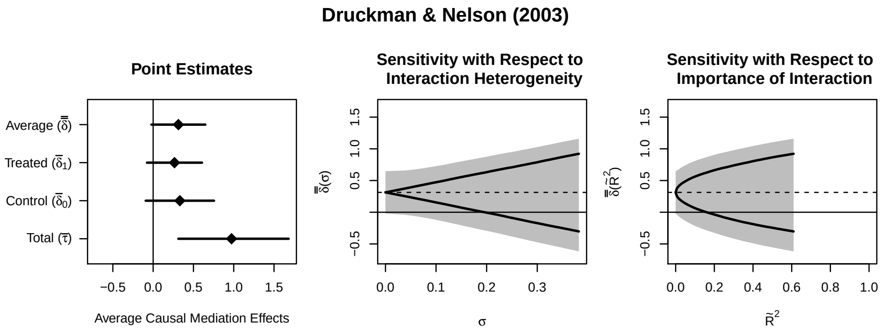
  - Implies an upper bound on σ: $0 < \sigma < \sqrt{\mathbb{V}(\eta_{3i}(T_i, M_i, W_i))/\mathbb{E}(T_i M_i^2)}$

Analysis under the Independence Assumption



- Weakly significant average indirect effects ($[0.025, 0.625]$), accounting for 28.6 percent of the total effect
- Moderate degree of sensitivity to the mediator exogeneity ($\bar{\bar{\delta}} = 0$ when $\rho = -0.43$ or $\tilde{R}^2_M \tilde{R}^2_Y = 0.078$)
- Concern (both theoretical and empirical) that the importance mechanism may be affected by the belief content mechanism

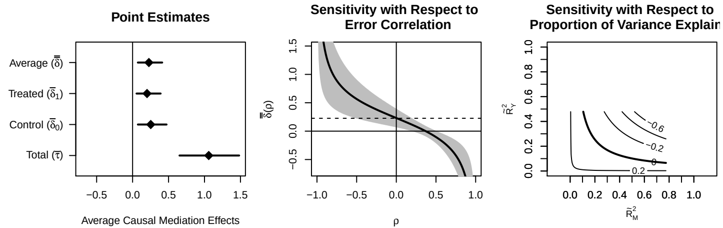
Analysis without the Independence Assumption



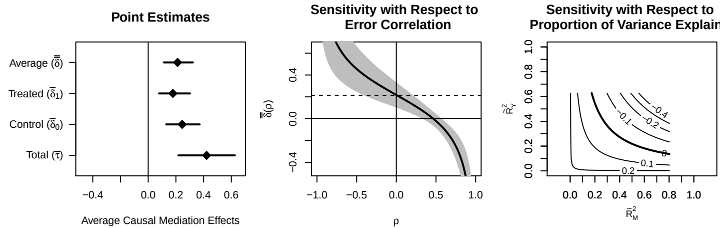
- The point estimate is similar with slightly wider CI ($[-0.021, 0.648]$)
- Lower bound on $\bar{\delta}$ equals zero when $\sigma = 0.195$, or 51% of its upper bound
- This translates to the interaction heterogeneity explaining 15.9% of the variance of the outcome variable

Analysis under the Independence Assumption

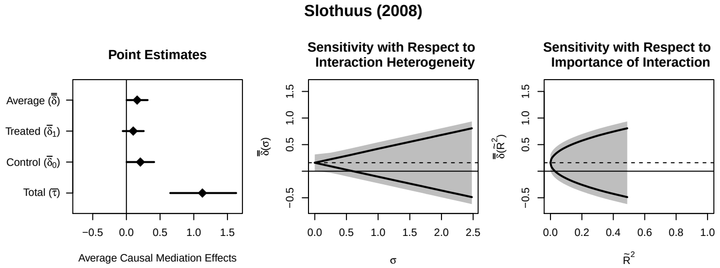
Slothuus (2008)

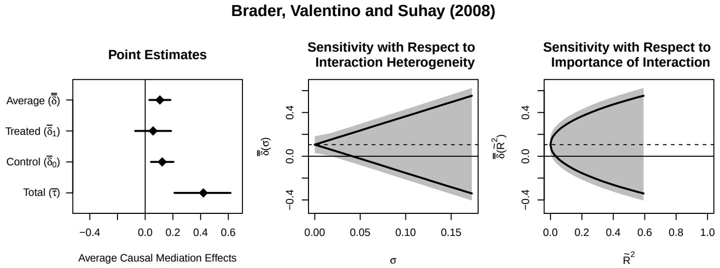


Brader, Valentino & Suhay (2008)



Analysis without the Independence Assumption





Extensions to New Experimental Designs

- The above analysis assumes (weak) sequential ignorability
- All pre- and post-treatment confounders are assumed to be observed
- Possible existence of unobserved confounders

- Randomized experiment to manipulate the primary mediator
- Natural experiments where the primary mediator is as-if random

- Parallel design:
  1. Randomize treatment
  2. Randomize both treatment and mediator
- Parallel encouragement design:
  - imperfect manipulation of the mediator
  - a randomized instrument for the mediator

Parallel Design

- Semi-parametric random coefficient linear model:

$$\begin{aligned} M_i(t) &= \alpha_2 + \beta_{2i} t + \epsilon_{2i} \\ Y_i(t,m) &= \alpha_3 + \beta_{3i} t + \gamma_i m + \kappa_i tm + \epsilon_{3i}, \end{aligned}$$

- Quantities of interest:

$$\begin{aligned} \bar{\delta}(t) &= \beta_1 - \bar{\zeta}(1-t) \\ \bar{\zeta}(t) &= \beta_3 + (\alpha_2 + \beta_2 t)\kappa + \rho_t \sigma \sqrt{\mathbb{V}(M_i \mid T_i = t, D_i = 0)} \end{aligned}$$

- Sensitivity analysis via ρ_t and σ

Parallel Encouragement Design

- Mediator model changes to

$$M_i(t, z) = \alpha_2 + \beta_{2i} t + \lambda_i z + \theta_i tz + \epsilon_{2i}$$

where z represents the value of randomized encouragement

- Outcome model stays identical to that for parallel design

$$Y_i(t, m) = \alpha_3 + \beta_{3i} t + \gamma_i m + \kappa_i tm + \epsilon_{3i},$$

- Two-stage least squares model
- Sensitivity analysis via ρ_{tz} and σ

Implementation via R Package **mediation**

An example syntax:

```
## pre-treatment covariates
Xnames <- c("age", "educ", "gender", "income")
## fit the model
m.med <- multimed(outcome = "immigr", med.main = "emo"
                  med.alt = "p_harm", treat = "treat",
                  covariates = Xnames,
                  data = framing, sims = 1000)
## summary
summary(m.med)
## point estimate under homogenous interaction
plot(m.med, type = "point")
## sensitivity analysis based on R2
plot(m.med, type = "R2-total")
```

For the parallel design, set `design = "parallel"` in `multimed()`

Concluding Remarks and Future Research

- Causal mediation analysis with multiple mediators is complicated!
- Critical issue: relationships among mediators
  1. causal ordering
  2. causal dependence
- (Sequential) ignorability is not sufficient:
  - Randomization of mediator does not solve the problem
  - Importance of heterogeneous treatment
  - Treatment-mediator interaction
- What explains heterogenous interaction effects?
- Can we adjust for those factors when designing and analyzing your study?
- Much methodological work remains to be done:
  - causal mediation in multi-level settings
  - causal mediation in longitudinal settings