

# POL 345: Quantitative Analysis and Politics

## Precept 7

### Week 8 (Verzani Chapter 7: 7.1-7.3)

In this precept, we cover the following new material:

- Using `rt()` to take random draws from a  $t$  distribution
- Using `pt()` to calculate the proportion of observations from a  $t$  distribution that take values greater than or equal to a specified value (equal to or less if `lower.tail` is specified as false)
- Using `qt()` to calculate percentiles from a  $t$  distribution
- Using `xaxt = "n"` to alter x-axis labels.

## 1 Confidence Intervals Using the Student's $t$ Distribution

The Student's  $t$  distribution is very similar to the normal distribution especially when the degrees of freedom is large.

- The function `rt(n, df)` will create a vector of length `n` containing independent, random draws from a  $t$  distribution with the specified degrees of freedom, `df`.
- The function `pt(q, df, lower.tail=TRUE)` will take in a vector of values, `q`, and report the proportion of all observations we would expect to witness having values equal to `q` or lower, given that the distribution is  $t$  with the specified `df`. Choosing `lower.tail=FALSE` will produce the proportion of values *greater* than `q`.
- The `qt(p, df)` function returns the 100`p`-th percentile of the  $t$  distribution with degrees of freedom `df`. Setting `lower.tail` to `FALSE` will return the 100(1 - `p`)-th percentile instead.
- Suppose that we have a sample of size  $n$  randomly drawn from a normally-distributed population with unknown variance. Under this assumption, the  $Z$ -score

$$Z = \frac{\bar{X} - \mathbb{E}(X)}{\text{s.e.}}$$

*exactly* follows the Student's  $t$  distribution with degrees of freedom  $n - 1$ . This statistic is called the  $t$  statistic. Thus, calculating the confidence interval when using a  $t$  distribution is similar to using a normal distribution. The only difference is that we use the commands associated with the  $t$  distribution rather than the normal distribution. Here, we calculate the 95% confidence intervals based on the normal and  $t$  distributions.

```

> samp.mean <- 5 ## sample mean
> st.dev <- 2 ## standard deviation
> n <- 20 ## sample size
> MoE.n <- qnorm(0.975)*st.dev/sqrt(n) ## Margin of error
> lower.n <- samp.mean - MoE.n
> upper.n <- samp.mean + MoE.n
> c(lower.n, upper.n) ## Confidence interval

[1] 4.12348 5.87652

> upper.n-lower.n ## Width of confidence interval

[1] 1.75305

> MoE.t <- qt(0.975, df = 20-1)*st.dev/sqrt(n) ## Margin of error
> lower.t <- samp.mean - MoE.t
> upper.t <- samp.mean + MoE.t
> c(lower.t, upper.t) ## Confidence interval

[1] 4.06397 5.93603

> upper.t-lower.t ## Slightly wider interval than under the normal distribution

[1] 1.87206

```

## The Democratic Peace

Beginning in the 1970s, scholars of International Relations found a renewed interest in the Democratic Peace Theory, as first proposed in Kant's *Perpetual Peace*. In short, the theory proposes that due to their affinity — similar norms, interests, and institutions — democracies are less likely to go to war with one another. We use data from the Polity IV project, the `dempeace.RData` file available on Blackboard, to examine hypotheses related to the Democratic Peace theory. The dataset describes dyadic relationships — each observation represents a conflict that occurs between two nations. The dataset covers the years surrounding the end of the Cold War. Our variables of interest are as follows:

- `politya` - Polity score for State A
  - `polityb` - Polity score for State B
  - `dispute` - Indicator variable that takes a value of 1 if the dyad is at war and a value of 0 if not
1. To begin, we need to classify each dyad. To do this, we adopt the conventional classification in International Relations: states are classified as democracies if their Polity score is 6 or above. States with scores between 5 and  $-5$  are classified as anocracies (societies with weak or non-existent central authorities). States with a Polity score of  $-6$  or below are classified as autocracies. Create an affinity variable coded as “Democratic” if both states are democracies, “Autocratic” if both states are autocracies, and “Mixed” otherwise.

2. Calculate the average number of disputes among the three types of dyads. Calculate the 99% confidence level for each. In a single figure, plot each estimate with the corresponding confidence intervals. Briefly interpret the results.

A variant of the Democratic Peace Theory—the Liberal Peace Theory—maintains that the observed peace between democracies emerges due to their higher likelihood to trade with one another, as compared to democracies trading with autocracies or autocracies trading with one another. Use the the `libpeace.RData` file, which includes the following variables:

- `politya` - Polity score for State A
  - `polityb` - Polity score for State B
  - `trade` - Total bilateral trade (logged) between State A and State B
3. As above, create an affinity variable coded as “Democratic”, “Autocratic”, or “Mixed”, according to the conventional classification in International Relations.
  4. Calculate the mean of dyadic trade among (1) democratic dyads, (2) autocratic dyads, and (3) mixed dyads. Calculate and interpret the 99% confidence intervals for each estimator. As above, in a single figure, plot the point estimates as solid circles along with the 99% confidence intervals, which should be represented as vertical lines. Briefly discuss the results.