POL502: Problem Set 3

Due October 7

1. Suppose x is a real number and $\epsilon > 0$. Prove that $(x - \epsilon, x + \epsilon)$ is a neighborhood of each of its members; in other words, if $y \in (x - \epsilon, x + \epsilon)$, then there is $\delta > 0$ such that $(y - \delta, y + \delta) \in (x - \epsilon, x + \epsilon)$.

2. Use the definition of convergence to prove that each of the following sequences converges:

   (a) $\{5 + \frac{1}{n}\}_{n=1}^{\infty}$

   (b) $\{2^{-n}\}_{n=1}^{\infty}$

3. Give an alternate proof of Theorem 1 (Uniqueness of Limit) along the following lines. Choose $\epsilon > 0$. There is N_1 such that $|a_n - A| < \frac{\epsilon}{2}$ for $n \geq N_1$, and there is N_2 such that $|a_n - B| < \frac{\epsilon}{2}$ for $n \geq N_2$. Use the triangle inequality to show that this implies that $|A - B| < \epsilon$. Argue that $A = B$.

4. Prove that the sequence $\{\frac{2n+1}{n}\}_{n=1}^{\infty}$ is Cauchy without using the result that every convergent sequence is Cauchy.

5. Give an example of a set with exactly two accumulation points.