Causal Interaction in High Dimension

Naoki Egami    Kosuke Imai

Princeton University

Talk at College of William and Mary

October 9, 2015
Interaction Effects and Causal Heterogeneity

1. Moderation
   - How do treatment effects vary across individuals?
   - Who benefits from (or is harmed by) the treatment?
   - Interaction between treatment and pre-treatment covariates

2. Causal interaction
   - What aspects of a treatment are responsible for causal effects?
   - What combination of treatments is efficacious?
   - Interaction between treatment variables

3. Individualized treatment regimes
   - What combination of treatments is optimal for a given individual?
Causal Interaction in High Dimension

- High dimension = many treatments, each having multiple levels

- A motivating application: Conjoint analysis (Hainmueller et al. 2014)
  - survey experiments to measure immigration preferences
  - a representative sample of 1,396 American adults
  - each respondent evaluates 5 pairs of immigrant profiles
  - gender^2, education^7, origin^{10}, experience^4, plan^4, language^4, profession^{11}, application reason^3, prior trips^5
  - Over 1 million treatment combinations!
  - What combinations of immigrant characteristics make them preferred?

- Too many treatment combinations \(\rightsquigarrow\) Need for an effective summary
- Interaction effects play an essential role
Two Interpretations of Causal Interaction

1. **Conditional effect interpretation:**
   - Does the effect of one treatment change as we vary the value of another treatment?
   - Does the effect of being black change depending on whether an applicant is male or female?
   - Useful for testing moderation among treatments

2. **Interactive effect interpretation:**
   - Does a combination of treatments induce an *additional effect* beyond the sum of separate effects attributable to each treatment?
   - Does being a black female induce an additional effect beyond the effect of being black and that of being female?
   - Useful for finding efficacious treatment combinations in high dimension
An Illustration in the $2 \times 2$ Case

- Two binary treatments: $A$ and $B$
- Potential outcomes: $Y(a, b)$ where $a, b \in \{0, 1\}$
- Conditional effect interpretation:
  \[
  \left[ Y(1, 1) - Y(0, 1) \right] - \left[ Y(1, 0) - Y(0, 0) \right]
  \]
  effect of $A$ when $B = 1$
  effect of $A$ when $B = 0$
  \[\rightsquigarrow\] requires the specification of moderator

- Interactive effect interpretation:
  \[
  \left[ Y(1, 1) - Y(0, 0) \right] - \left[ Y(1, 0) - Y(0, 0) \right] - \left[ Y(0, 1) - Y(0, 0) \right]
  \]
  effect of $A$ and $B$
  effect of $A$ when $B = 0$
  effect of $B$ when $A = 0$
  \[\rightsquigarrow\] requires the specification of baseline condition

- The same quantity but two different interpretations
Difficulty of the Conventional Approach

- **Lack of invariance** to the baseline condition
  \[ \implies \text{Inference depends on the choice of baseline condition} \]

- **3 \times 3** example:
  - Treatment \( A \in \{a_0, a_1, a_2\} \) and Treatment \( B \in \{b_0, b_1, b_2\} \)
  - Regression model with the baseline condition \((a_0, b_0)\):

    \[
    E(Y | A, B) = 1 + a_1^* + a_2^* + b_2^* + a_1^*b_2^* + 2a_2^*b_2^* + 3a_2^*b_1^* \]

    - Interaction effect for \((a_2, b_2)\) > Interaction effect for \((a_1, b_2)\)

    - Another equivalent model with the baseline condition \((a_0, b_1)\):

    \[
    E(Y | A, B) = 1 + a_1^* + 4a_2^* + b_2^* + a_1^*b_2^* - a_2^*b_2^* - 3a_2^*b_0^* \]

    - Interaction effect for \((a_2, b_2)\) < Interaction effect for \((a_1, b_2)\)
    - Interaction effect for \((a_2, b_1)\) is zero under the second model
    - All interaction effects with at least one baseline value are zero
Empirical Illustration: Lack of Invariance

- Linear regression with main effects and two-way interactions
- Baseline: *lowest* levels of job experiences and education

<table>
<thead>
<tr>
<th>Job experience</th>
<th>None</th>
<th>4th grade</th>
<th>8th grade</th>
<th>High school</th>
<th>Two-year college</th>
<th>College</th>
<th>Graduate</th>
</tr>
</thead>
<tbody>
<tr>
<td>None (baseline)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1–2 years</td>
<td>0</td>
<td>0.009</td>
<td>-0.019</td>
<td>-0.032</td>
<td>0.100</td>
<td>-0.044</td>
<td>-0.064</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.063)</td>
<td>(0.063)</td>
<td>(0.063)</td>
<td>(0.064)</td>
<td></td>
<td>(0.063)</td>
</tr>
<tr>
<td>3–5 years</td>
<td>0</td>
<td>0.016</td>
<td>0.056</td>
<td>0.165</td>
<td>0.107</td>
<td>0.010</td>
<td>0.117</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.063)</td>
<td>(0.064)</td>
<td>(0.064)</td>
<td>(0.064)</td>
<td></td>
<td>(0.063)</td>
</tr>
<tr>
<td>&gt; 5 years</td>
<td>0</td>
<td>-0.050</td>
<td>0.126</td>
<td>0.042</td>
<td>0.058</td>
<td>-0.094</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.064)</td>
<td>(0.064)</td>
<td>(0.063)</td>
<td>(0.064)</td>
<td></td>
<td>(0.064)</td>
</tr>
</tbody>
</table>
The Effects of Changing the Baseline Condition

- Same linear regression but different baseline
- Baseline: *highest* levels of job experiences and education

<table>
<thead>
<tr>
<th>Job experience</th>
<th>None</th>
<th>4th grade</th>
<th>8th grade</th>
<th>High school</th>
<th>Two-year college</th>
<th>College</th>
<th>Graduate</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>0.015</td>
<td>0.065</td>
<td>-0.111</td>
<td>-0.027</td>
<td>-0.043</td>
<td>0.109</td>
<td>0</td>
</tr>
<tr>
<td>(0.064)</td>
<td>(0.062)</td>
<td>(0.064)</td>
<td>(0.061)</td>
<td>(0.063)</td>
<td>(0.063)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1–2 years</td>
<td>0.078</td>
<td>0.138</td>
<td>-0.066</td>
<td>0.006</td>
<td>0.120</td>
<td>0.129</td>
<td>0</td>
</tr>
<tr>
<td>(0.064)</td>
<td>(0.062)</td>
<td>(0.062)</td>
<td>(0.061)</td>
<td>(0.062)</td>
<td>(0.062)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3–5 years</td>
<td>-0.102</td>
<td>-0.036</td>
<td>-0.172</td>
<td>0.021</td>
<td>-0.054</td>
<td>0.002</td>
<td>0</td>
</tr>
<tr>
<td>(0.062)</td>
<td>(0.062)</td>
<td>(0.063)</td>
<td>(0.062)</td>
<td>(0.061)</td>
<td>(0.062)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&gt; 5 years</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

(baseline)
The Contributions of the Paper

1 Problems of the conventional approach:
   • Lack of invariance to the choice of baseline condition
   • Difficulty of interpretation for higher-order interaction

2 Solution: Average Marginal Treatment Interaction Effect
   • invariant to baseline condition
   • same, intuitive interpretation even for high dimension
   • simple estimation procedure

3 Reanalysis of the immigration survey experiment
Two-way Causal Interaction

- Two factorial treatments:
  
  \[ A \in A = \{ a_0, a_1, \ldots, a_{D_A-1} \} \]
  
  \[ B \in B = \{ b_0, b_1, \ldots, b_{D_B-1} \} \]

- Assumption: Full factorial design
  1. Randomization of treatment assignment
  \[ \{ Y(a_\ell, b_m) \}_{a_\ell \in A, b_m \in B} \perp \perp \{ A, B \} \]
  2. Non-zero probability for all treatment combination
  \[ \Pr(A = a_\ell, B = b_m) > 0 \text{ for all } a_\ell \in A \text{ and } b_m \in B \]

- Fractional factorial design not allowed
  1. Use a small non-zero assignment probability
  2. Focus on a subsample
  3. Combine treatments
Non-Interaction Effects of Interest

1. Average Treatment Combination Effect (ATCE):
   - Average effect of treatment combination \((A, B) = (a_\ell, b_m)\) relative to the baseline condition \((A, B) = (a_0, b_0)\)
     \[
     \tau(a_\ell, b_m; a_0, b_0) \equiv \mathbb{E}\{Y(a_\ell, b_m) - Y(a_0, b_0)\}
     \]
   - Which treatment combination is most efficacious?

2. Average Marginal Treatment Effect (AMTE; Hainmueller et al. 2014):
   - Average effect of treatment \(A = a_\ell\) relative to the baseline condition \(A = a_0\) averaging over the other treatment \(B\)
     \[
     \psi(a_\ell, a_0) \equiv \int_B \mathbb{E}\{Y(a_\ell, B) - Y(a_0, B)\} dF(B)
     \]
   - Which treatment is effective on average?
The Conventional Approach to Causal Interaction

- **Average Treatment Interaction Effect (ATIE):**
  \[ \xi(a_\ell, b_m; a_0, b_0) \equiv \mathbb{E}\{Y(a_\ell, b_m) - Y(a_0, b_m) - Y(a_\ell, b_0) + Y(a_0, b_0)\} \]

- **Conditional effect interpretation:**
  \[ \mathbb{E}\{Y(a_\ell, b_m) - Y(a_0, b_m)\} - \mathbb{E}\{Y(a_\ell, b_0) - Y(a_0, b_0)\} \]
  
  Effect of \( A = a_\ell \) when \( B = b_m \)  
  Effect of \( A = a_\ell \) when \( B = b_0 \)

- **Interactive effect interpretation:**
  \[ \tau(a_\ell, b_m; a_0, b_0) - \mathbb{E}\{Y(a_\ell, b_0) - Y(a_0, b_0)\} - \mathbb{E}\{Y(a_0, b_m) - Y(a_0, b_0)\} \]
  
  ATCE  
  Effect of \( A = a_\ell \) when \( B = b_0 \)  
  Effect of \( B = b_m \) when \( A = a_0 \)

- **Estimation:** Linear regression with interaction terms
Ineffectiveness of Interaction Plot in High Dimension

Problem: it does not plot interaction effects themselves
### ATIE is Sensitive to the Choice of Baseline Condition

<table>
<thead>
<tr>
<th>Job experience</th>
<th>None</th>
<th>4th grade</th>
<th>8th grade</th>
<th>High school</th>
<th>Two-year college</th>
<th>College</th>
<th>Graduate</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1–2 years</td>
<td>0</td>
<td>0.009</td>
<td>–0.019</td>
<td>–0.032</td>
<td>0.100</td>
<td>–0.044</td>
<td>–0.064</td>
</tr>
<tr>
<td>3–5 years</td>
<td>0</td>
<td>0.016</td>
<td>0.056</td>
<td>0.165</td>
<td>0.107</td>
<td>0.010</td>
<td>0.117</td>
</tr>
<tr>
<td>&gt; 5 years</td>
<td>0</td>
<td>–0.050</td>
<td>0.126</td>
<td>0.042</td>
<td>0.058</td>
<td>–0.094</td>
<td>0.015</td>
</tr>
</tbody>
</table>
ATIE is Sensitive to the Choice of Baseline Condition

<table>
<thead>
<tr>
<th>Job experience</th>
<th>None</th>
<th>4th grade</th>
<th>8th grade</th>
<th>High school</th>
<th>Two-year college</th>
<th>College</th>
<th>Graduate</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>0.015</td>
<td>0.065</td>
<td>-0.111</td>
<td>-0.027</td>
<td>-0.043</td>
<td>0.109</td>
<td>0</td>
</tr>
<tr>
<td>1–2 years</td>
<td>0.078</td>
<td>0.138</td>
<td>-0.066</td>
<td>0.006</td>
<td>0.120</td>
<td>0.129</td>
<td>0</td>
</tr>
<tr>
<td>3–5 years</td>
<td>-0.102</td>
<td>-0.036</td>
<td>-0.172</td>
<td>0.021</td>
<td>-0.054</td>
<td>0.002</td>
<td>0</td>
</tr>
<tr>
<td>&gt; 5 years</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Lack of Invariance to the Baseline Condition

- Comparison between two ATIEs should not be affected by the choice of baseline conditions
- We prove that the ATIEs are *neither interval or order invariant*

- **Interval invariance:**

\[
\xi(a_\ell, b_m; a_0, b_0) - \xi(a_{\ell'}, b_{m'}; a_0, b_0) = \xi(a_\ell, b_m; a_\tilde{\ell}, b_\tilde{m}) - \xi(a_{\ell'}, b_{m'}; a_\tilde{\ell}, b_\tilde{m}),
\]

- **Order invariance:**

\[
\xi(a_\ell, b_m; a_0, b_0) \geq \xi(a_{\ell'}, b_{m'}; a_0, b_0) \iff \xi(a_\ell, b_m; a_\tilde{\ell}, b_\tilde{m}) \geq \xi(a_{\ell'}, b_{m'}; a_\tilde{\ell}, b_\tilde{m}).
\]
The New Causal Interaction Effect

- **Average Marginal Treatment Interaction Effect (AMTIE):**

  \[ \pi(a_\ell, b_m; a_0, b_0) \equiv \tau(a_\ell, b_m; a_0, b_0) - \psi(a_\ell, a_0) - \psi(b_m, b_0) \]

  - ATCE of \((A, B) = (a_\ell, b_m)\)
  - AMTE of \(a_\ell\)
  - AMTE of \(b_m\)

- Interactive effect interpretation: additional effect induced by \(A = a_\ell\) and \(B = b_m\) together beyond the separate effect of \(A = a_\ell\) and that of \(B = b_m\)

- Compare this with ATIE:

  \[ \tau(a_\ell, b_m; a_0, b_0) - \mathbb{E}\{Y(a_\ell, b_0) - Y(a_0, b_0)\} - \mathbb{E}\{Y(a_0, b_m) - Y(a_0, b_0)\} \]

  - ATCE
  - Effect of \(A = a_\ell\) when \(B = b_0\)
  - Effect of \(B = b_m\) when \(A = a_0\)

- We prove that the **AMTIEs** are both *interval and order invariant*

- The **AMTIEs** do depend on the distribution of treatment assignment
  - specified by one’s experimental design
  - motivated by the target population
AMTIE is Invariant to the Choice of Baseline Condition

<table>
<thead>
<tr>
<th>Job experience</th>
<th>None</th>
<th>4th grade</th>
<th>8th grade</th>
<th>High school</th>
<th>Two-year college</th>
<th>College</th>
<th>Graduate</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>0</td>
<td>−0.004</td>
<td>−0.028</td>
<td>−0.035</td>
<td>−0.031</td>
<td>0.012</td>
<td>−0.010</td>
</tr>
<tr>
<td>1–2 years</td>
<td>−0.001</td>
<td>−0.001</td>
<td>−0.025</td>
<td>−0.040</td>
<td>0.024</td>
<td>−0.009</td>
<td>−0.044</td>
</tr>
<tr>
<td>3–5 years</td>
<td>−0.040</td>
<td>−0.019</td>
<td>−0.042</td>
<td>0.031</td>
<td>−0.026</td>
<td>−0.022</td>
<td>0.024</td>
</tr>
<tr>
<td>&gt; 5 years</td>
<td>−0.014</td>
<td>−0.031</td>
<td>0.041</td>
<td>−0.011</td>
<td>−0.021</td>
<td>−0.036</td>
<td>−0.024</td>
</tr>
</tbody>
</table>
AMTIE is Invariant to the Choice of Baseline Condition

<table>
<thead>
<tr>
<th>Job experience</th>
<th>None</th>
<th>4th grade</th>
<th>8th grade</th>
<th>High school</th>
<th>Two-year college</th>
<th>College</th>
<th>Graduate</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>0.024</td>
<td>0.020</td>
<td>−0.004</td>
<td>−0.011</td>
<td>−0.007</td>
<td>0.036</td>
<td>0.014</td>
</tr>
<tr>
<td>1–2 years</td>
<td>0.023</td>
<td>0.023</td>
<td>−0.001</td>
<td>−0.016</td>
<td>0.048</td>
<td>0.015</td>
<td>−0.020</td>
</tr>
<tr>
<td>3–5 years</td>
<td>−0.016</td>
<td>0.005</td>
<td>−0.018</td>
<td>0.055</td>
<td>−0.002</td>
<td>0.002</td>
<td>0.048</td>
</tr>
<tr>
<td>&gt; 5 years</td>
<td>0.010</td>
<td>−0.007</td>
<td>0.065</td>
<td>0.013</td>
<td>0.003</td>
<td>−0.012</td>
<td>0</td>
</tr>
</tbody>
</table>
The Relationships between the ATIE and the AMTIE

1. The AMTIE is a linear function of the ATIEs:

\[
\pi(a_\ell, b_m; a_0, b_0) = \xi(a_\ell, b_m; a_0, b_0) - \sum_{a \in A} \Pr(A_i = a) \xi(a, b_m; a_0, b_0) \\
- \sum_{b \in B} \Pr(B_i = b) \xi(a_\ell, b; a_0, b_0)
\]

2. The ATIE is also a linear function of the AMTIEs:

\[
\xi(a_\ell, b_m; a_0, b_0) = \pi(a_\ell, b_m; a_0, b_0) - \pi(a_\ell, b_0; a_0, b_0) - \pi(a_0, b_m; a_0, b_0)
\]

- Absence of causal interaction:
  All of the AMTIEs are zero if and only if all of the ATIEs are zero

- The AMTIEs can be estimated by first estimating the ATIEs
Higher-order Causal Interaction

- $J$ factorial treatments: $\mathbf{T} = (T_1, \ldots, T_J)$

- Assumptions:
  1. Full factorial design
     \[ Y(t) \perp \perp \mathbf{T} \quad \text{and} \quad \Pr(\mathbf{T} = t) > 0 \quad \text{for all } t \]
  2. Independent treatment assignment
     \[ T_j \perp \perp \mathbf{T}_{-j} \quad \text{for all } j \]

- Assumption 2 is not necessary for identification but considerably simplifies estimation

- We are interested in the $K$-way interaction where $K \leq J$

- We extend all the results for the 2-way interaction to this general case
Difficulty of Interpreting the Higher-order ATIE

- Generalize the 2-way ATIE by marginalizing the other treatments $T^{1:2}$

$$\xi_{1:2}(t_1, t_2; t_{01}, t_{02}) \equiv \int \mathbb{E} \left\{ Y(t_1, t_2, T^{1:2}) - Y(t_{01}, t_2, T^{1:2}) ight. $$

$$\left. - Y(t_1, t_{02}, T^{1:2}) + Y(t_{01}, t_{02}, T^{1:2}) \right\} dF(T^{1:2})$$

- In the literature, the 3-way ATIE is defined as

$$\xi_{1:3}(t_1, t_2, t_3; t_{01}, t_{02}, t_{03})$$

$$\equiv \xi_{1:2}(t_1, t_2; t_{01}, t_{02} \mid T_3 = t_3) - \xi_{1:2}(t_1, t_2; t_{01}, t_{02} \mid T_3 = t_{03})$$

2-way ATIE when $T_3 = t_3$

2-way ATIE when $T_3 = t_{03}$

- Higher-order ATIEs are similarly defined sequentially

- This representation is based on the conditional effect interpretation

- Problem: the conditional effect of conditional effects!
Interactive Interpretation of the Higher-order ATIE

- We show that the higher-order ATIE also has an interactive effect interpretation
- Example: 3-way ATIE, $\xi_{1:3}(t_1, t_2, t_3; t_{01}, t_{02}, t_{03})$, equals

$$
\tau_{1:3}(t_1, t_2, t_3; t_{01}, t_{02}, t_{03})
\begin{array}{l}
\text{ATCE}
\end{array}
$$

$$
- \{\xi_{1:2}(t_1, t_2; t_{01}, t_{02} | T_3 = t_{03}) + \xi_{2:3}(t_2, t_3; t_{02}, t_{03} | T_1 = t_{01})
+ \xi_{1:3}(t_1, t_3; t_{01}, t_{03} | T_2 = t_{02})\} \quad \text{sum of 2-way conditional ATIEs}
$$

$$
- \{\tau_1(t_1, t_{02}, t_{03}; t_{01}, t_{02}, t_{03}) + \tau_2(t_{01}, t_2, t_{03}; t_{01}, t_{02}, t_{03})
+ \tau_3(t_{01}, t_{02}, t_3; t_{01}, t_{02}, t_{03})\} \quad \text{sum of (1-way) ATCEs}
$$

- Problems:
  1. Lower-order *conditional* ATIEs rather than lower-order ATIEs are used
  2. K-way ATCE $\neq$ sum of all K-way and lower-order ATIEs
  3. (We prove) Lack of invariance to the baseline conditions
The $K$-way Average Marginal Treatment Interaction Effect

- **Definition:** the difference between the ATCE and the sum of lower-order AMTIEs

- **Interactive effect interpretation**

- **Example:** 3-way AMTIE, $\pi_{1:3}(t_1, t_2, t_3; t_{01}, t_{02}, t_{03})$, equals

$$
\underbrace{\pi_{1:3}(t_1, t_2, t_3; t_{01}, t_{02}, t_{03})}_{\text{ATCE}} - \left\{ \pi_{1:2}(t_1, t_2; t_{01}, t_{02}) + \pi_{2:3}(t_2, t_3; t_{02}, t_{03}) + \pi_{1:3}(t_1, t_3; t_{01}, t_{03}) \right\} \quad \text{sum of 2-way AMTIEs}
$$

$$
- \left\{ \psi(t_1; t_{01}) + \psi(t_2; t_{02}) + \psi(t_3; t_{03}) \right\} \quad \text{sum of (1-way) AMTEs}
$$

- **Properties:**
  1. $K$-way ATCE = the sum of all $K$-way and lower-order AMTIEs
  2. Interval and order invariance to the baseline condition
  3. Derive the relationships between the AMTIEs and ATIEs for any order
Empirical Analysis of the Immigration Survey Experiment

- 5 factors (gender, education, origin, experience, plan)
  1. full factorial design assumption
  2. computational tractability

- Matched-pair conjoint analysis: randomly choose one profile
- Binary outcome: whether a profile is selected
- Model with one-way, two-way, and three-way interaction terms
- $p = 1,575$ and $n = 6,980$
- Curse of dimensionality $\Rightarrow$ sparcity assumption
- Support vector machine with a lasso constraint (Imai & Ratkovic, 2013)
- Under-identified model that includes baseline conditions
- 99 non-zero and 1,476 zero coefficients
- Cross-validation for selecting a tuning parameter
- FindIt: Finding heterogeneous treatment effects
- **Range of AMTIEs:** importance of each factor and factor interaction

- **Sparcity-of-effects principle**

- Gender appears to play a significant role in three-way interactions
- Exploration of level interactions
- \( \text{origin} \times \text{experience} \) interaction
- Baseline: India, None
- Only relative magnitude matters
- Little interaction for European origin
- Similar pattern for Mexico and Phillipines as well as Sudan and Somalia
Decomposing the Average Treatment Combination Effect

- **Two-way effect example** \((\text{origin} \times \text{experience})\):

\[
\tau(\text{Somalia, 1-2 years}; \text{India, None}) = -3.74 = \psi(\text{Somalia}; \text{India}) + \psi(1-2\text{years}; \text{None}) + \pi(\text{Somalia, 1-2years}; \text{India, None}) - 5.14 + 5.12 - 3.72
\]

- **Three-way examples** \((\text{education} \times \text{gender} \times \text{origin})\):

\[
\tau(\text{Graduate, Male, India}; \text{Graduate, Female, India}) = 7.46 = \psi(\text{Male}; \text{Female}) + \pi(\text{Graduate, Male}; \text{Graduate, Female}) - 0.77 - 0.34 + \pi(\text{Male, India}; \text{Female, India}) + \pi(\text{Graduate, Male, India}; \text{Graduate, Female, India}) 1.56 + 7.01
\]
\[
\tau(\text{High school, Male, Germany; High school, Female, Germany}) \quad -11.52 \\
(\text{n} = 41; \quad \text{n} = 56)
\]

\[
\psi(\text{Male; Female}) + \pi(\text{High school, Male; High school, Female}) - 0.77
\quad + 0.67
\]

\[
\pi(\text{Male, Germany; Female, Germany}) - 3.34
\quad + \pi(\text{High school, Male, Germany; High school, Female, Germany}) - 6.74
\]
Concluding Remarks

- Interaction effects play an essential role in causal heterogeneity
  1. moderation
  2. causal interaction

- Two interpretations of causal interaction
  1. conditional effect interpretation (problematic in high dimension)
  2. interactive effect interpretation

- Average Marginal Treatment Interaction Effect
  1. interactive effect in high-dimension
  2. invariant to baseline condition
  3. enables effect decomposition
  4. $\Rightarrow$ effective analysis of interactions in high-dimension

- Estimation challenges in high dimension
  1. group lasso, hierarchical interaction
  2. post-selection inference


Send comments and suggestions to negami@Princeton.Edu or kimai@Princeton.Edu